

Probability and Statistics for Elementary and Middle School Teachers

A Staff Development Training Program To Implement the 2001 Virginia Standards of Learning

Revised December 2004

Division of Instruction
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Introduction

The revised *Probability and Statistics for Elementary and Middle School Teachers* is a staff development training program designed to assist teachers in implementing the 2001 Virginia Standards of Learning for mathematics. This staff development program provides a sample of meaningful and engaging activities correlated to the probability and statistics strand of the grades K-5 and grades 6–8 mathematics Standards of Learning.

The purpose of the staff development program is to enhance teachers' content knowledge and their use of instructional strategies for teaching the probability and statistics Standards of Learning. Teachers will receive intensive training on ways to (1) gather data, and (2) represent, analyze, and interpret data to guide instruction and classroom assessment. Through explorations, problem solving, and hands-on experiences, teachers will engage in discussions and strategies that address:

- · formulating questions and conducting investigations;
- gathering data and using tools from simple tallying methods to the development of good surveys and methods of observation;
- representing data in a variety of tables, charts, graphs and plots (including line plots, stem and-leaf plots, and box-and-whisker plots);
- developing strategies for analyzing and interpreting data, making inferences, observing trends, drawing conclusions, and making predictions; and
- assessing students data analysis skills and knowledge.

Through these activities, it is anticipated that teachers will develop new techniques that are sure to enhance student achievement in their classroom.

Designed to be presented by teacher trainers, this staff development program includes directions for the trainer, as well as the black line masters for handouts. In some instances, related student activities are included. Trainers should adapt the materials to best fit the needs of their audience; adding materials that may be more appropriate for their audience and eliminating materials that have been used in previous training sessions. Trainers are encouraged to use graphing utilities and other technology, as appropriate. All materials in this document may be duplicated and distributed as desired for use in Virginia.

The training program is organized into five three-hour modules that may be offered by school divisions for teacher licensure renewal points or for a one-credit graduate course, when university credit can be arranged.



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GLOSSARY

Average See mean.

Axes See x-axis and y-axis.

Bar graph A graph that uses parallel horizontal or vertical bars to represent

counts for several categories. One bar is used for each category, with the length of the bar representing the count for that category.

Box-and-whisker

plot

A graph showing how a set of data clusters around the middle (median) and shows the distribution of data in each quartile.

Circle graph A circular graph that shows the relationship of the parts to the

whole.

Coordinates An ordered pair of numbers used to locate a point in a plane.

Coordinate system (coordinate plane)

A two-dimensional system of intersecting horizontal and vertical

number lines, used to locate points.

Fundamental (Basic)
Counting Principle

A computational procedure used to determine the number of possible arrangements of several objects. It is the product of the number of ways each object can be chosen individually (e.g., the possible arrangements of four shirts, two pants, and three shoes

is 4 x 2 x 3 or 24).

Event An outcome or set of outcomes of an experiment or situation.

e.g., rolling a 3 or higher is one possible event produced by a dice

roll.

Experiment In probability, any activity involving chance, such as a dice roll.

Experimental probability

A probability based on the statistical results of an experiment.

Fair games Games where all players have the same odds of winning.

Histogram A type of bar graph where the categories are equal ranges of

numbers.

Independent events The event in which the outcome of one event does not affect the

probability of the subsequent event.

Line graph A graph that uses a line to show how data changes over time.

Line plot A plot, using stacked x 's, showing the distribution of values in a

data set.



Mean The sum of the values in a data set divided by the number of

values. Also known as the average.

Median The middle value in a data set when the values are arranged in

order.

Mode The value(s) that occur most often in a data set.

Negative relationship Two data sets have a negative relationship when the data values

in one set increase as the values in the other decrease.

Odds The ratio of a number of ways an event can happen to the

number of ways it cannot.

Outcome One way an experiment or situation could turn out.

Outlier A value widely separated from the others in a data set. Any value

that lies more than 1.5 IQR units below the lower quartile or 1.5

IQR units above the upper quartile.

Positive relationship Two data sets have a positive relationship when their data values

increase or decrease together.

Probability The likelihood of an event occurring.

Quadrants The four regions determined by the axes of a coordinate plane.

Range (in statistics) The difference between the least and greatest numbers in a data

set.

Rate A ratio showing how quantities with different units are related.

Example: <u>72 dollars</u> 8 hours

Sample Space All the possible outcomes of an experiment.

Scale (graphical) A system of marks in a given order and at specific intervals.

Scatterplot A graph showing paired data values as points.

Simulation A model of a real world situation.

Stem-and-leaf plot A table showing the distribution of values in a data set by splitting

each value into a "stem" and a "leaf".

Theoretical probability

The ratio of the number of ways an event can happen to the total

number of possible outcomes.



Tree diagram A branching diagram showing all possible outcomes for a given

experiment.

Trend A clear direction in a line graph suggesting how the data will

behave in the future.

x-axis The horizontal number line in a coordinate plane.

x-coordinate The first number in an ordered pair.

y-axis The vertical number line in a coordinate plane.

y-coordinate The second number in an ordered pair.



THE BIG IDEAS OF STATISTICS POSING QUESTIONS Session 1

Topic	Activity	Page	Related SOL	Activity	Materials
	Name	Number		Sheets	
The Big Ideas of	Sandwich Problem	2	K.14, K.15, K.16, 1.18,	Sandwich Problem	
Statistics	Fiobleiii		1.19, 2.23,		
Statistics				Narrative, Sandwich	
			2.24, 3.21,		
			3.22, 3.23, 4.19,4.20,	Problem Graph	
	Whytoro	5	5.16, 5.17,		Blank
	Why are	5	5.18, 5.19,	Readings,	
	Probability		· · · · · · · · · · · · · · · · · · ·	Probability	transparencies,
	and Statistics Important?		6.18, 6.19, 6.20,7.14,	and Statistics Strands	overhead pens
	The Big	19	7.15, 7.16,	2 Graphic	
	Ideas of		7.17, 7.18,	Organizer	
	Statistics		8.11, 8.12,	Sheets, SOL	
			8.13	Probability	
				and Statistics	
				Strand	
				Grades K-8	
	What are the	25		Probability	
	Goals of the			and Statistics	
	Course?			Goals	
Posing	Sixth Grade	29	1.19, 2.23,	Questions for	
Questions	Mystery Data		3.22, 4.20,	Sixth Grade	
			5.18, 6.18,	Mystery	
			7.18, 8.12	Data, Graphs	
				for Sixth	
				Grade	
				Mystery	
				Data, Graph	
				A, Graph B,	
				Graph C	
	Posing	36	1.19, 2.23,	Posing	Chart paper,
	Questions		3.22, 6.18,	Questions	markers, tape
			7.18, 8.12		



Activity: Sandwich Problem (Warm-Up)

Format: Large Group

Objectives: Participants will develop an appreciation for graphical

representations of data and the need for statistics.

Related SOL: All in the Probability and Statistics Strand

Materials: Sandwich Problem Narrative Activity Sheet, and Sandwich

Problem Graph Activity Sheet

Time Required: 10 minutes

Directions:

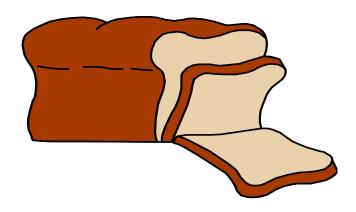
1. Without informing the participants, break them into two groups (front of the audience versus back of the audience).

- 2. Distribute the two sandwich problem activity sheets FACE DOWN; distribute the graphical representation of the sandwich data to one group; and the narrative version of the data to the other group.
- 3. Tell the groups that this is a test on the sandwich data and that you are going to keep track of the people who raise their hand first to answer the questions. Then ask them to turn over their papers and respond to the following questions. Keep track of those who answer first, expecting that those with the graphical answer will respond first. Ask the following three questions. Call on the first person who raises a hand to answer the question.
 - 1. What sandwich was preferred more by people than any other sandwich?
 - 2. What sandwich types were preferred by only two people?
 - 3. What sandwich type did Oliver prefer?
- 4. After reinforcing that one part of the room was doing better than the others in answering the questions, show the entire group a copy of both types of data. This is a good place to begin the discussion of why statistics is important in this "information age" which can be found in Session I Activity 2.
- 5. Distribute the extra copies of activity sheets so that each participant has a copy of both the graph and narrative sandwich problem activity sheets.



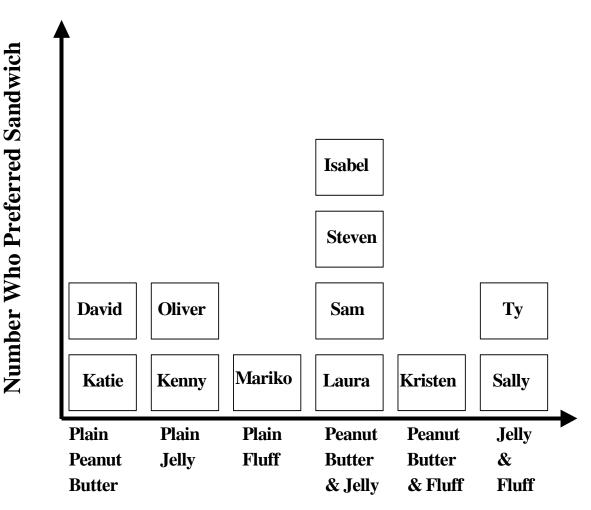
The Lunch Bunch's Favorites

Laura had peanut butter and jelly. Kenny had plain jelly. Oliver also had plain jelly. Katie and David had plain peanut butter. Oh, I forgot to mention that Steven, Isabel, and Sam also had peanut butter and jelly. Kristen had peanut butter and fluff. Mariko had plain fluff while Sally and Ty had jelly and fluff.

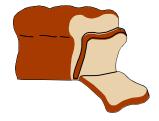




The Lunch Bunch's Favorites



Sandwich Types





Activity: Why are Probability and Statistics Important?

Format: Whole group, Mini-Lecture

Objectives: Participants will understand the rationale for including

probability and statistics topics in the K-8 curriculum based on

the NCTM Standards.

Related SOL: All in the Probability and Statistics Standards of Learning

Materials: Activity Sheets containing Excerpts 1-5 from the NCTM

Principles and Standards for School Mathematics "Why are Probability and Statistics Important in the K-8 Curriculum?"; Strands of Probability and Statistics in the Principles and

Standards for School Mathematics (2000): blank

transparencies, overhead pens

Time Required: 45 minutes

Directions:

- Use a modified form of the "Jigsaw" cooperative learning procedure to answer the question "Why are Probability and Statistics Important in the K-8 Curriculum?" Use the NCTM Principles and Standards for School Mathematics excerpts on Probability and Statistics as the resource for this "Jigsaw."
- 2. In jigsaw, five groups of participants will be responsible for teaching each other the material. The NCTM resource is divided into 5 expert areas. Each group should be assigned one excerpt and each team member should read the material first. At a signal, all participants on a team should get together and elect a leader and a reporter for their team. The leader should facilitate a discussion to identify the important points in their excerpt to answer the question "Why are Probability and Statistics Important in the K-8 Curriculum?" The reporter should record the points mentioned by team members.
- 3. The instructor should then ask the reporter for each team to report on the team's discussion and record the ideas on a blank overhead transparency. The instructor may wish to summarize the participant's remarks and add any of his or her own before concluding this activity.



NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS Principles and Standards for School Mathematics - Resource for Discussion on "Why are Statistics Important in the K-8 Curriculum?"

EXCERPT 1:

Standard 5: Data Analysis and Probability Instructional programs from Prekindergarten through grade 12 should enable all students to

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- Select and use appropriate statistical methods to analyze data;
- Develop and evaluate inferences and predictions that are based on data; and
- Understand and apply basic concepts of probability.

Elaboration: Pre-K-12

The Data Analysis and Probability Standard recommends that students formulate questions that can be answered using data and addresses what is involved in gathering and using the data wisely. Students should learn how to collect data, organize their own or others' data, and display the data in graphs and charts that will be useful in answering their questions. This Standard also includes learning some methods for analyzing data and some ways of making inferences and conclusions from data. The basic concepts and applications of probability are also addressed, with an emphasis on the way that probability and statistics are related.

The amount of data available to help make decisions in business, politics, research, and everyday life is staggering: Consumer surveys guide the development and marketing of products. Polls help determine political-campaign strategies, and experiments are used to evaluate the safety and efficacy of new medical treatments. Statistics are often misused to sway public opinion on issues or to misrepresent the quality and effectiveness of commercial products. Students need to know about data analysis and related aspects of probability in order to reason statistically—skills necessary to becoming informed citizens and intelligent consumers.

The increased curricular emphasis on data analysis proposed in these Standards is intended to span the grades rather than to be reserved for the middle grades and secondary school, as is common in many countries. NCTM's 1989 *Curriculum and Evaluation Standards for School Mathematics* introduced standards in statistics and probability at all grade bands; a number of organizations have developed instructional materials and professional



development programs to promote the teaching and learning of these topics. Building on this base, these Standards recommend a strong development of the strand, with concepts and procedures becoming increasingly sophisticated across the grades so that by the end of high school students have a sound knowledge of elementary statistics. To understand the fundamentals of statistical ideas, students must work directly with data. The emphasis on working with data entails students' meeting new ideas and procedures as they progress through the grades rather than revisiting the same activities and topics. The data and statistics strand allows teachers and students to make a number of important connections among ideas and procedures from number, algebra, measurement, and geometry. Work in data analysis and probability offers a natural way for students to connect mathematics with other school subjects and with experiences in their daily lives.

In addition, the processes used in reasoning about data and statistics will serve students well in work and in life. Some things children learn in school seem to them predetermined and rule bound. In studying data and statistics, they can also learn that solutions to some problems depend on assumptions and have some degree of uncertainty. The kind of reasoning used in probability and statistics is not always intuitive, and so students will not necessarily develop it if it is not included in the curriculum.



NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS Principles and Standards for School Mathematics - Resource for Discussion on "Why is Statistics Important in the K-8 Curriculum?"

EXCERPT 2:

Standard 5: Data Analysis and Probability Instructional programs from Prekindergarten through grade 12 should enable all students to

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- Select and use appropriate statistical methods to analyze data;
- Develop and evaluate inferences and predictions that are based on data; and
- Understand and apply basic concepts of probability.

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

Because young children are naturally curious about their world, they often raise questions such as, How many? How much? What kind? or Which of these? Such questions often offer opportunities for beginning the study of data analysis and probability. Young children like to design questions about things close to their experience—What kind of pets do classmates have? What are children's favorite kinds of pizza? As students move to higher grades, the questions they generate for investigation can be based on current issues and interests. Students in grades 6–8, for example, may be interested in recycling, conservation, or manufacturers' claims. They may pose questions such as, Is it better to use paper or plastic plates in the cafeteria? or Which brand of batteries lasts longer? By grades 9–12, students will be ready to pose and investigate problems that explore complex issues.

Young children can devise simple data-gathering plans to attempt to answer their questions. In the primary grades, the teacher might help frame the question or provide a tally sheet, class roster, or chart on which data can be recorded as they are collected. The "data" might be real objects, such as children's shoes arranged in a bar graph or the children themselves arranged by interest areas. As students move through the elementary grades, they should spend more time planning the data collection and evaluating how well their methods worked in getting information about their questions. In the middle grades, students should work more with data that have been gathered by others or generated by simulations. By grades 9–12, students should understand the various purposes of surveys, observational studies, and experiments.



A fundamental idea in prekindergarten through grade 2 is that data can be organized or ordered and that this "picture" of the data provides information about the phenomenon or question. In grades 3–5, students should develop skill in representing their data, often using bar graphs, tables, or line plots. They should learn what different numbers, symbols, and points mean. Recognizing that some numbers represent the values of the data and others represent the frequency with which those values occur is a big step. As students begin to understand ways of representing data, they will be ready to compare two or more data sets. Books, newspapers, the World Wide Web, and other media are full of displays of data, and by the upper elementary grades, students ought to learn to read and understand these displays. Students in grades 6–8 should begin to compare the effectiveness of various types of displays in organizing the data for further analysis or in presenting the data clearly to an audience. As students deal with larger or more-complex data sets, they can reorder data and represent data in graphs quickly, using technology so that they can focus on analyzing the data and understanding what they mean.



NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS Principles and Standards for School Mathematics - Resource for Discussion on "Why is Statistics Important in the K-8 Curriculum?"

EXCERPT 3:

Standard 5: Data Analysis and Probability Instructional programs from Prekindergarten through grade 12 should enable all students to

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- Select and use appropriate statistical methods to analyze data;
- Develop and evaluate inferences and predictions that are based on data; and
- Understand and apply basic concepts of probability.

Select and use appropriate statistical methods to analyze data.

Although young children are often most interested in their own piece of data on a graph (I have five people in my family), putting all the students' information in one place draws attention to the set of data. Later, students should begin to describe the set of data as a whole. Although this transition is difficult (Konold forthcoming), students may, for example, note that "more students come to school by bus than by all the other ways combined." By grades 3–5, students should be developing an understanding of aggregated data. As older students begin to see a set of data as a whole, they need tools to describe this set. Statistics such as measures of center or location (e.g., mean, median, mode), measures of spread or dispersion (range, standard deviation), and attributes of the shape of the data become useful to students as descriptors. In the elementary grades, students' understandings can be grounded in informal ideas, such as middle, concentration, or balance point (Mokros and Russell 1995). With increasing sophistication in secondary school, students should choose particular summary statistics according to the questions to be answered.

Throughout the school years, students should learn what it means to make valid statistical comparisons. In the elementary grades, students might say that one group has more or less of some attribute than another. By the middle grades, students should be quantifying these differences by comparing specific statistics. Beginning in grades 3–5 and continuing in the middle grades, the emphasis should shift from analyzing and describing one set of data to comparing two or more sets (Konold forthcoming). As they move through the middle grades into high school, students will need new tools, including histograms, stem-and-leaf plots, box plots, and scatterplots, to identify similarities and differences among data sets. Students also need tools to investigate association and bivariate



data, including scatterplots and fitted lines in grades 6–8 and residuals and correlation in grades 9–12.



NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS Principles and Standards for School Mathematics - Resource for Discussion on "Why is Statistics Important in the K-8 Curriculum?"

EXCERPT 4:

Standard 5: Data Analysis and Probability Instructional programs from Prekindergarten through grade 12 should enable all students to

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- Select and use appropriate statistical methods to analyze data;
- Develop and evaluate inferences and predictions that are based on data; and
- Understand and apply basic concepts of probability.

Develop and evaluate inferences and predictions that are based on data

Central elements of statistical analysis—defining an appropriate sample, collecting data from that sample, describing the sample, and making reasonable inferences relating the sample and the population—should be understood as students move through the grades. In the early grades, students are most often working with census data, such as a survey of each child in the class about favorite kinds of ice cream. The notion that the class can be viewed as a sample from a larger population is not obvious at these grades. Upper elementary and early middle-grades students can begin to develop notions about statistical inference, but developing a deep understanding of the idea of sampling is difficult (Schwartz et al. 1998). Research has shown that students in grades 5—8 expect their own judgment to be more reliable than information obtained from data (Hancock, Kaput, and Goldsmith 1992). In the later middle grades and high school, students should address the ideas of sample selection and statistical inference and begin to understand that there are ways of quantifying how certain one can be about statistical results.

In addition, students in grades 9–12 should use simulations to learn about sampling distributions and make informal inferences. In particular, they should know that basic statistical techniques are used to monitor quality in the workplace. Students should leave secondary school » with the ability to judge the validity of arguments that are based on data, such as those that appear in the press.



NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS Principles and Standards for School Mathematics - Resource for Discussion on "Why is Statistics Important in the K-8 Curriculum?"

EXCERPT 5:

Standard 5: Data Analysis and Probability Instructional programs from Prekindergarten through grade 12 should enable all students to

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- Select and use appropriate statistical methods to analyze data;
- Develop and evaluate inferences and predictions that are based on data; and
- Understand and apply basic concepts of probability.

Understand and apply basic concepts of probability

A subject in its own right, probability is connected to other areas of mathematics, especially number and geometry. Ideas from probability serve as a foundation to the collection, description, and interpretation of data.

In prekindergarten through grade 2, the treatment of probability ideas should be informal. Teachers should build on children's developing vocabulary to introduce and highlight probability notions, for example, We'll probably have recess this afternoon, or It's unlikely to rain today. Young children can begin building an understanding of chance and randomness by doing experiments with concrete objects, such as choosing colored chips from a bag. In grades 3-5 students can consider ideas of chance through experiments—using coins, dice, or spinners—with known theoretical outcomes or through designating familiar events as impossible, unlikely, likely, or certain. Middle-grades students should learn and use appropriate terminology and should be able to compute probabilities for simple compound events, such as the number of expected occurrences of two heads when two coins are tossed 100 times. In high school, students should compute probabilities of compound events and understand conditional and independent events. Through the grades, students should be able to move from situations for which the probability of an event can readily be determined to situations in which sampling and simulations help them quantify the likelihood of an uncertain outcome.

Many of the phenomena that students encounter, especially in school, have predictable outcomes. When a fair coin is flipped, it is equally likely to come up heads or tails. Which outcome will result on a given flip is uncertain—even if ten flips in a row have resulted in heads, for many people it is counterintuitive that



the eleventh flip has only a 50 percent likelihood of being tails. If an event is random and if it is repeated many, many times, then the distribution of outcomes forms a pattern. The idea that individual events are not predictable in such a situation but that a pattern of outcomes can be predicted is an important concept that serves as a foundation for the study of inferential statistics.



NCTM Focus	NCTM Focus Areas 3-5	NCTM Focus Areas 6-8	
Areas K - 2	In grades 3-5, all students	In grades 6-8, all students	
In grades pre-K-2,	should-	should-	
all students should-			
Formulate	Formulate questions that	Formulate questions that	
questions that	can be addressed with	can be addressed with	
can be	data and collect,	data and collect, organize,	
addressed with	organize, and display	and display relevant data	
data and collect,	relevant data to answer	to answer them	
organize, and	them	- formulate questions	
display relevant data to answer	design investigations to	 formulate questions, design studies, and 	
them	address a question and	collect data about a	
	consider how data-	characteristic shared by	
• pose	collection methods	two populations or	
questions and	affect the nature of the	different characteristics	
gather data	data set;	within one population;	
about themselves	 collect data using observations, surveys, 	 select, create, and use appropriate graphical 	
and their	and experiments;	representations of data,	
surroundings;	 represent data using 	including histograms,	
sort and	tables and graphs such	box plots, and	
classify	as line plots, bar	scatterplots.	
objects	graphs, and line graphs;		
according to their attributes	recognize the		
and organize	differences in		
data about the	representing categorical and numerical data.		
objects;	and namenous data.		
 represent data 			
using concrete			
objects,			
pictures, and			
graphs.			



NCTM Focus	NCTM Focus Areas 3-5	NCTM Focus Areas 6-8	
Areas	In grades 3-5, all students	In grades 6-8, all	
K – 2	should-	students should-	
In grades pre-K-2,			
all students should-			
Select and use	Select and use	Select and use	
appropriate	appropriate statistical	appropriate statistical	
statistical	methods to analyze data	methods to analyze data	
methods to			
describe parts of the data and the set of data as a whole to determine what the data show.	 describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed; use measures of center, focusing on the median, and understand what each does and does not indicate about the data set; compare different representations of the same data and evaluate how well each representation shows important aspects of the data. 	 find, use, and interpret measures of center and spread, including mean and interquartile range; discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots. 	



NCTM Focus Areas	NCTM Focus Areas 3-5	NCTM Focus Areas 6-8
PreK - 2	In grades 3 - 5, all	In grades 6 - 8, all students
In grades pre-K-2, all students should-	students should-	should-
Develop and	Develop and evaluate	Develop and evaluate
evaluate	inferences and	inferences and predictions
inferences and	predictions that are	that are based on data
predictions that	based on data	
discuss events related to students' experiences as likely or unlikely.	propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions.	 use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken; make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit; use conjectures to formulate new questions and plan new studies to answer them.



NCTM Focus Areas 3-5 In grades 3-5, all students should-	NCTM Focus Areas 6-8 In grades 6-8, all students should-
Understand and apply basic concepts of probability • describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible; • predict the probability of outcomes of simple experiments and test the predictions; • understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.	 Understand and apply basic concepts of probability understand and use appropriate terminology to describe complementary and mutually exclusive events; use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations; compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.



Activity: The Big Ideas of Statistics

Format: Small Group

Objectives: Participants will discuss the meaning of statistics and identify

the big ideas that constitute statistics in the K-8 Standards of

Learning.

Related SOL: All in the Probability and Statistics Standards of Learning

Materials: Graphic Organizer Sheet "The Big Ideas of Statistics", grade

level copies of the Standards of Learning for Probability and Statistics, The Process of Statistical Investigation Graphic

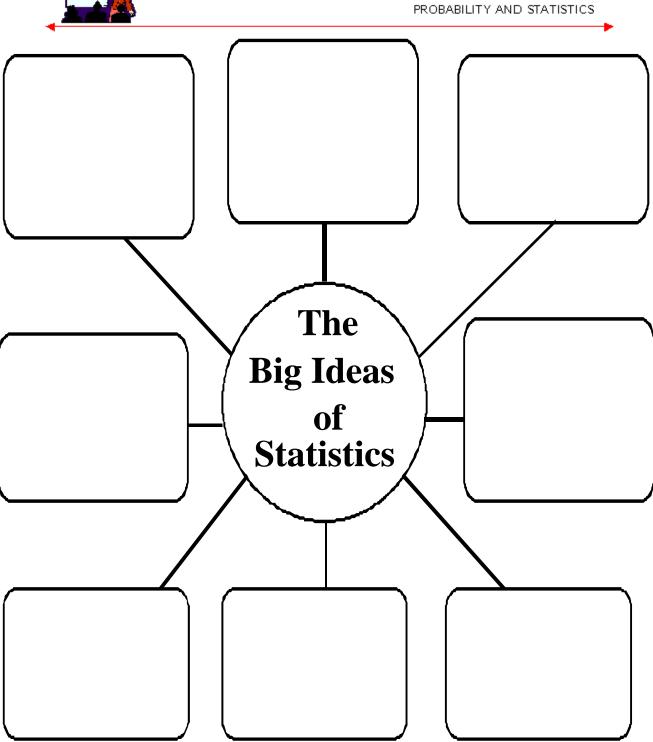
Organizer Sheet

Time Required: 15 minutes

Directions:

- 1. Organize participants into small groups of 4 (preferably with different grade levels represented). Each group receives one copy of the graphic organizer sheet, The Big Ideas of Statistics.
- 2. Pose the question: What are the "Big Ideas" or the Key Concepts that should be taught in grades K-5 (or Grades 6-8 depending upon your audience) Probability and Statistics? Have participants refer to the Standards of Learning to look for key words and ideas. Have a group recorder write the words they associate with statistics into the graphic organizer. Give groups about 5 minutes to discuss.
- 3. Have groups contribute ideas in a large group discussion. Record ideas in a webbing fashion to show how ideas connect on The Big Ideas of Statistics. Discuss each briefly and give examples. The big ideas include: pose a problem, collect data, organize data, represent the data graphically, analyze the data, and interpret the data.
- 4. Discuss the process of statistical investigation and show The Process of Statistical Investigation Graphic Organizer Sheet.







Virginia Standards of Learning

Probability and Statistics Strand Grades K - 5

- K.14 The student will gather data relating to familiar experiences by counting and tallying.
- K.15 The student will display objects and information, using objects, graphs, pictorial graphs, and tables.
- K.16 The student will investigate and describe the results of dropping a two-colored counter or using a multicolored spinner.
- 1.18 The student will investigate, identify, and describe various forms of data collection in his/her world (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream), using tables, picture graphs, and object graphs.
- 1.19 The student will interpret information displayed in a picture or object graph using the vocabulary: more, less, fewer, greater than, less than, and equal to.
- 2.23 The student will read, construct, and interpret a simple picture and bar graph.
- 2.24 The student will record data from experiments using spinners and colored tiles/cubes and use the data to predict which of two events is more likely to occur if the experiment is repeated.
- 3.21 The student, given grid paper, will a) collect and organize data on a given topic of his/her choice, using observations, measurements, surveys, or experiments; and b) construct a line plot, a picture graph, a bar graph to represent the results. Each graph will include an appropriate title and key.
- 3.22 The student will read and interpret data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.
- 3.23 The student will investigate and describe the concept of probability as chance, and list possible results of a given situation.
- 4.19 The student will a) predict the likelihood of outcomes of a simple event, using the terms certain, likely, unlikely, impossible; and b) determine the probability of a given simple event, using concrete materials.
- 4.20 The student will collect, organize, and display data in line and bar graphs with scale increments of one or greater than one and use the display to interpret the results, draw conclusions, and make predictions.



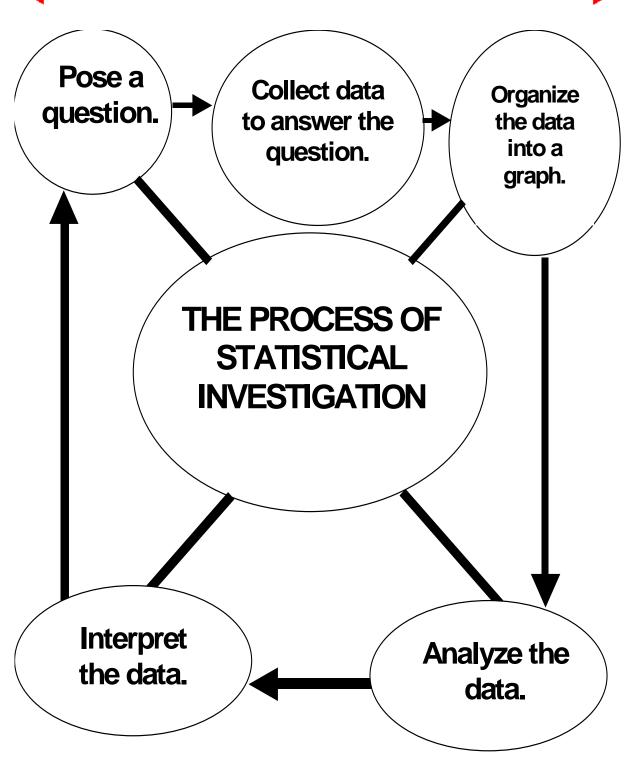
- 5.17 The student will a) solve problems involving the probability of a single event by using tree diagrams or by constructing a sample space representing all possible results; b) predict the probability of outcomes of simple experiments, representing it with fractions or decimals from 0 to 1, and test the prediction; and c) create a problem statement involving probability based on information from a given problem situation. Students will not be required to solve the created problem statement.
- 5.18 The student will, given a problem situation, collect, organize, and display a set of numerical data in a variety of forms, using bar graphs, stem-and-leaf plots, and line graphs, to draw conclusions and make predictions.
- 5.19 The student will find the mean, median, mode, and range of a set of data.



Probability and Statistics Strand Grades 6 - 8

- 6.18 The student, given a problem situation, will collect, analyze, display, and interpret data in a variety of graphical methods, including a) line, bar, and circle graphs; b) stem-and-leaf plots; and c) box-and-whisker plots. Circle graphs will be limited to halves, fourths, and eighths.
- 6.19 The student will describe the mean, median, and mode as measures of central tendency, describe the range, and determine their meaning for a set of data.
- 6.20 The student will a) make a sample space for selected experiments and represent it in the form of a list, chart, picture, or tree diagram; and b) determine and interpret the probability of an event occurring from a given sample space and represent the probability as a ratio, decimal, or percent, as appropriate for the given situation.
- 7.14 The student will investigate and describe the difference between the probability of an event found through simulation versus the theoretical probability of that same event.
- 7.15 The student will identify and describe the number of possible arrangements of several objects, using a tree diagram or the Fundamental (Basic) Counting Principle.
- 7.16 The student will create and solve problems involving the measures of central tendency (mean, median, mode) and the range of a set of data.
- 7.17 The student, given a problem situation, will collect, analyze, display, and interpret data, using a variety of graphical methods, including a) frequency distributions; b) line plots; c) histograms; d) stem-and-leaf plots; e) box-and-whisker plots; and f) scattergrams.
- 7.18 The student will make inferences, conjectures, and predictions based on analysis of a set of data.
- 8.11 The student will analyze problem situations, including games of chance, board games, or grading scales, and make predictions, using knowledge of probability.
- 8.12 The student will make comparisons, predictions, and inferences, using information displayed in frequency distributions; box-and-whisker plots; scattergrams; line, bar, circle, and picture graphs; and histograms.
- 8.13 The student will use a matrix to organize and describe data.







Activity: What Are the Goals of the Course?

Format: Whole group, Mini-Lecture

Objectives: Participants will review the goals of the institute and discuss

their present use of probability and statistics in their classroom.

Related SOL: Probability and Statistics Standards of Learning

Materials: Probability and Statistics Goals Activity Sheet

Time Required: 15 minutes

Directions:

1. Review each of the goals indicated in the goals of the course statement.

Ask questions to find out what teachers are presently doing in their teaching of probability and statistics to glean a sense of their knowledge and use of the forthcoming content.



PROBABILITY AND STATISTICS GOALS

The teacher will:

- 1. Formulate a question and understand its relationship to the problem under study and the variables that will be used for the statistical investigation.
- 2. Sort and classify objects according to similar attributes to identify "variables" (i.e., categories, independent variable, etc.) contained within a set of data.
- 3. Demonstrate various techniques to collect data including:
 - surveys
 - observations
 - questionnaires
 - polls
 - interviews
 - examining past records
 - experimentation using instruments
 - simulations.
- 4. Use tabulations such as tallies, counters, and counting to record the collected data.
- 5. Demonstrate various techniques to organize data including:
 - pictures
 - lists
 - charts
 - tables
 - tree diagrams
 - matrices
 - stem-and-leaf plots
 - ordered pairs.
- 6. Determine the most effective graphical method to display a given set of data.
- 7. Display data using graphical representations including:
 - object graphs
 - bar graphs
 - circle graphs
 - picture graphs
 - line graphs
 - line plots



- frequency distributions
- box-and-whisker plots
- stem-and-leaf plots
- the coordinate plane
- scattergrams
- histograms.
- 8. Determine and interpret the measures of central tendency and the range.
- 9. Analyze the data and interpret the results of an investigation. Consider the elements such as:
 - common features
 - unique features
 - similarities and differences
 - the relationships between the variables
 - cause and effect
 - generalizations
 - predictions
 - averaging.
- 10. Address the following questions:
 - What do the results mean?
 - What does the graph show?
 - What would happen "if"?
 - What do you expect?
 - Is there a change?
 - Is there a relationship?
 - Is there a pattern?
 - What conclusions can you draw?
 - How do your results compare to your predictions?
- 11. Investigate and determine the probability of a given simple event by representing all possible results of the given event using:
 - · concrete materials
 - lists
 - charts
 - tables
 - tree diagrams
 - constructing a sample space
 - the Fundamental (Basic) Counting Principle.



- 12. Express the probability of a given simple event in an appropriate form, including:
 - ratios
 - decimals
 - percents.
- 13. Analyze and interpret the results of a probability problem including:
 - predicting which of two events is more likely
 - investigating the differences between the probability of an event found through simulation (experimental probability) versus theoretical probability
 - · determining whether the experiment is fair.



Activity: Sixth Grade Mystery Data

Format: Small Groups; Whole Group

Objectives: Participants will develop an understanding of the relationship

between the question and the analysis of the data.

Related SOL: 1.19, 2.23, 3.22, 4.20, 5.18, 6.18, 7.18, 8.12

<u>Materials</u>: Copies of Sixth Grade Mystery Data, Copies of Questions,

Copies of Graphs A, B, and C

Time Required: 30 minutes

Directions:

1. Divide the participants into small groups of four to five. Give each group a copy of the Sixth Grade Mystery Data and a copy of the questions to be answered. Tell them they have 15 minutes to answer the questions and to discuss their solutions.

- 2. After the small groups have completed the task, have the entire group share their solutions and how they arrived at those solutions. Focus the discussion on the relationship of the question to the data.
- 3. Discuss Graph A (Ice Cream Preferences). Have participants share questions about this graph that could be asked of K-2 students.
- 4. Discuss Graph B (Number of Cavities). Have participants share questions about this graph that could be asked of students in grades 3-5.
- 5. Discuss Graph C (Relationship of Height to Age). Have participants share questions about this graph that could be asked of students in grades 6-8.



Sixth Grade Mystery Data

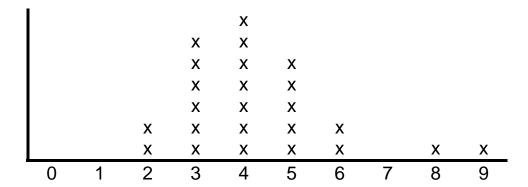
Look at the graphs on the next pages. Each graph shows something about a classroom of sixth graders.

- 1. Which of the five graphs do you think shows:
 - a. The number of cavities the sixth graders have?
 - b. The number of people in the sixth graders' families?
 - c. The ages of the sixth graders' mothers?
 - d. The heights of the sixth graders in inches?
- 2. Why do you think the graph you picked for d is the one that shows the heights of sixth graders? Why do you think the other graphs don't show the sixth graders' heights?
- 3. One of the graphs was not selected to answer question one above. What do you think this data display might represent? Why?

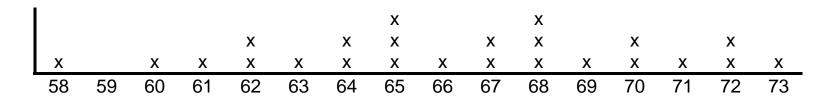


Sixth Grade Mystery Data

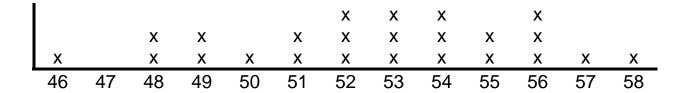
Graph 1



Graph 2

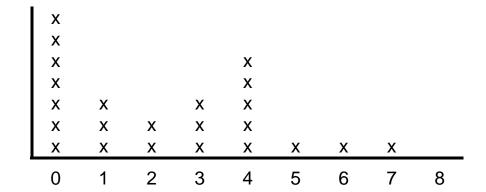


Graph 3

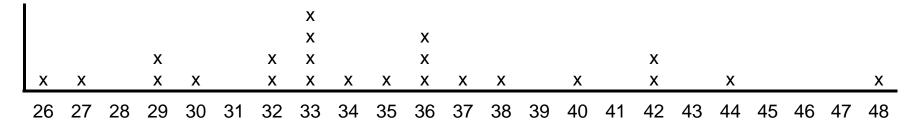




Graph 4



Graph 5





Graph A

Ice Cream Preferences

Vanilla











Chocolate







Mint Chip







Cookies & Cream









Other











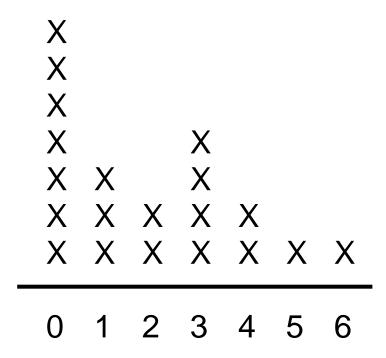


= 1 student



Graph B

Number of Cavities





Graph C

Relationship of Height to Age





Activity: Posing Questions

Format: Small Group; Large Group

Objectives: Participants will construct a variety of grade-appropriate

question stems that focus on ideas such as generalizing, comparing and contrasting, summarizing, describing,

interpreting, and predicting.

Related SOL: 1.19, 2.23, 3.22, 4.20, 5.18, 6.18, 7.18, 8.12

Materials: Chart paper and markers, tape, Posing Questions Activity

Sheet, Examples of Questions Activity Sheet

Time Required: 40 minutes

Directions:

- 1. Remind participants that, in the last activity, they looked at questions that helped focus on mystery data. Explain that, in this activity, the focus will be on designing questions to help students look beyond just "reading" a graph. Discuss the terms on the Posing Questions Activity Sheet. Have the whole group discuss the meanings of, and the differences between, the terms displayed. Be certain that the discussion includes the following:
 - Generalizing is finding the general or over-all character of a set of data, as opposed to specific characteristics, so that broad inferences can be made.
 - Comparing and contrasting is the discussion of the characteristics the data (or graphs) have in common as well as the differences.
 - Describing data means looking at the shape or trends in the data display.
 - Interpreting the data means looking at what is displayed in relationship to the question posed.
 - Summarizing data means looking at the main points seen in the data display and is a concise explanation of the results.
 - Predicting is the act of using the data to predict the results of additional sampling or extending the conclusions beyond the original sample to a larger group.
- 2. Divide the participants into grade-level groups (K, 1, 2, 3...). If the entire group is small or a particular grade level is underrepresented, it may be necessary to group participants (e.g., K-1, 2-3, 4-5, and 6-8). Give each group several sheets of chart paper and some markers.
- 3. Ask each small group to review the SOL for their grade level to review the types of data displays that are appropriate. Have them think about the kinds



of questions students might use to collect data and how those displays might look.

4.	Have the small groups generate grade-appropriate question stems that could
	be used by students to collect data. Share examples (use Activity Sheet with
	Examples) such as the following. A first grader might complete the stem,
	"What is your favorite?" to collect data to construct a picture or
	object graph. Older students might use the stems "Has there been a change
	over time?" or "Is there a relationship betweenand?"
	Have each group label the chart paper with the appropriate grade level, write
	their question stems on the chart paper, and post them on the wall.

- 5. Have participants do a museum walk. Each group moves around the room and reviews the grade-appropriate question stems of the other groups. The focus of the tour is to study the question stems generated for each grade level looking for common threads, and progression in the level of complexity through the grade levels.
- Tell the participants that this topic will be revisited as they complete other sessions. Question stems that can be used to generate the specific type of graph being studied will be added to their lists.
- 7. Reconvene the large group and have participants share what they learned about forming question stems at different grade levels.



Posing questions that focus on data and cause students to:

- generalize
- compare and contrast
- describe
- interpret
- summarize
- predict



Examples To Get You Started Posing questions that focus on data and cause students to:

generalize

How many M&M's could you expect to find in a fun-size bag of M&M's?

How many Blue M&M's could you expect to find in a fun-size bag of M&M's?

compare and contrast

What are the differences in rainfall between the city that you live in and the city you most want to visit?

describe

How many hours of TV are watched by the typical student in the classroom?

How long can the typical fifth grader hold their breath?

summarize

What is the typical foot length (to the nearest quarter inch) of students in a given grade (or in each of the grades from kindergarten through fifth grade)?

predict

What would you predict to be the differences in height (in inches) between the shortest and tallest within any classroom in the school?



COLLECTING AND ORGANIZING DATA MEASURES OF CENTRAL TENDENCY REPRESENTING DATA Session 2

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Collecting and Organizing Data	Collecting Data: Count the Ways	41	K.14, 1.18, 2.23, 3.21, 4.20, 5.18, 6.18, 7.18		Chart paper or transparency, markers
	Random Sampling	43	K.14, 1.18, 2.23, 3.21, 4.20, 5.18, 6.18, 7.18	Data Recording Sheet	Sticky notes, bag or basket
	Household Data	47	K.15, 1.18, 2.23, 3.21, 4.20, 5.18 6.18, 7.18, 8.13	Household Survey Data Sheet	Index cards with data organizer, chart paper, tape, markers
Measures of Central Tendency	Grab A Handful	50	5.19, 6.19, 7.16	Grab a Handful Recording Sheet	Linking cubes, sticky notes, container for cubes
Representing Data	What's Missing?	53	K.15, 1.19, 2.23, 3.22, 4.20, 5.18, 6.18,7.18, 8.12	Candy Graphs A, B, C	
	Object Graphs and Picture Graphs	58	K.15, 1.19, 2.23, 3.22, 4.20, 5.18, 6.18	Ideas for Graphing, T- shirt Pattern	Colored paper squares, linking cubes, scissors, markers, grid paper
	Attributes of Bar Graphs and Attributes of Polygons	62	2.23, 3.21, 3.22, 4.20, 5.18, 6.18, 8.12	Collection of Polygons Sheet, Shading of Polygons Graph	Scissors



Activity: Collecting Data - Count the Ways

Format: Small group

Objectives: Participants will discuss various methods of data collection.

Related SOL: K.14, 1.18, 2.23, 3.21, 4.20, 5.18, 6.18, 7.18

Materials: Chart paper (or transparency) to record ideas, Methods of

Collecting Data Activity Sheet

Time Required: 20 minutes

Background:

There are a variety of methods for collecting data including counting and tallying, many of which are familiar to children. Children may also have experience with giving or taking surveys. Discussion with students should explore difficulties that can be encountered with surveys. For example, a person

may be unwilling to answer a survey/guestionnaire/

poll/interview or they may be unwilling to answer the questions accurately. Also, the difficulties that occur in returning and

collecting surveys should be discussed.

Children should have an initial understanding of how the above variables can affect the outcome/results. While an interview is typically one-on-one, and the interviewer can somewhat control the conversation, the accuracy of the respondent's answer can still affect the results. Another method of data collection is to examine past records such as polls/surveys conducted, newspaper articles, and public records. However, accuracy of these records cannot be assured, as the method of data collection may be unknown.

Experiments using instruments such as thermometers, yardsticks, calculator-based labs (CBLs), and probes can provide data for use in the classroom.

Simulation can be used to understand natural fluctuations or variation in data. An example is to determine whether a spinner with 6 spaces is "fair". A fair spinner would produce each outcome (1-6) an equal number of times over many, many spins. For instance, if we spun the spinner 60 times, we would expect each outcome to occur approximately 10 times. Notice we say "approximately" because we expect some variation – for instance, maybe eight 1s, thirteen 2s, etc.



Directions:

- 1. Organize participants into small groups of four (preferably with different grade levels represented) to play a game.
- 2. Each group will need to select a recorder.
- 3. Explain to the participants that they will be working in groups to brainstorm as many ways as they can think of to collect data.
- 4. The catch to this is that they will only get points for their response if no other group in the room has that response. Therefore, they will want to try and think of as many unusual or uncommon methods as possible in addition to methods they believe other groups will think of.
- 5. Allow groups approximately five minutes to work.
- 6. When time is up, choose one group to begin the sharing process.
- 7. As the group leader reads the responses, the instructor will record them on the large chart paper or transparency. After each response, if no other group in the room has that idea written down, the group should record a +1 next to that method. If another group(s) has the method, all groups should cross that idea off of their paper.
- 8. When the first group is done sharing, the instructor should ask if any other groups have methods that were not read off by the first group. If so, repeat the above process as the next group shares any remaining methods.
- Sharing continues in this manner until the groups share all possible methods. Some methods that should appear on the list include counting, tallying, measurement, surveys, observations, questionnaires, polls, interviews, examining past records, simulations, and experiments.
- 10. The group with the most points at the end of this process wins the game.
- 11. Add to the list any other methods of data collection not thought of by the groups and make the list available during the session to add to by participants as appropriate.
- 12. Have participants take out their list of grade appropriate stems/questions (from the activities in "Posing Questions", Session I) and identify which of the above methods they would use to gather information to address their stems/questions.
- 13. Ask them, "When would you use these techniques?" "How would you adapt them to fit the needs of your students?"
- 14. Have participants copy the group list, or make copies available at end of session.



Activity: Random Sampling

Format: Large Group

Objective: Participants will develop an understanding of appropriate

methods of sampling and data collection to ensure that the data provides a representative, unbiased sample of the population.

Related SOL: K.14, 1.18, 2.23, 3.21, 4.20, 5.18, 6.18, 7.18

Materials: Data Recording Sheet, Biased and Unbiased Sampling

Methods Sheet, basket or bag from which to draw a sample,

sticky notes

Time Required: 45 minutes

Background: As presented in the previous activity, there are several methods

for collecting data. Most methods require the researcher to collect data from a population. In most circumstances, collecting information from every member of a population is impossible. Therefore, we collect data from a sample of the population and use the sample to make inferences about the population. Samples can be very accurate in describing the population characteristics. However, for samples to be accurate, they must represent the population. If we wanted to know how much TV middle school students watch, we would not just talk to boys from the eighth grade. This sample of the population only represents one portion of the population—male eighth graders. If the sample is not representative of the entire

population, the sample is considered biased because it does

not accurately reflect the population being studied.

Two methods that frequently generate biased data are judgment samples and convenience samples. Judgment samples are developed when the researcher uses his/her own judgment to determine what is representative of the population. This method often brings in the researcher's bias about what the population results should be. Convenience samples are developed when researchers select those who are easiest to reach for their sample.

The method used for developing a sample that truly represents the population is random sampling. Randomly selecting a sample does not mean haphazardly selecting members of the population. Rather, it means that each member has an equal



chance of being selected and each sample from the population has an equal chance of being selected. Random methods require that the researcher have an accurate list of the population. The researcher then selects the sample by numbering the list and generating random numbers or putting the names in a hat and drawing out names to be included in the sample. When generating the sample, sample size needs to be considered. Enough data needs to be collected to be sure that conclusions are accurate.

A key point to make through this activity is that all samples vary. Rarely will a researcher take two samples from the population and get the same results. Some students think of sample variation as bias. Bias is difficult to judge from one sample. Sampling methods that are biased will show their bias over several samples. For instance, we cannot prove that a coin is an unfair coin by flipping it once or even ten times. We might get more than five heads if we flip a coin only ten times. However, if we continue to flip the coin ten times and our results consistently show more than five heads, we can say the coin is biased. Over several trials, random, unbiased samples will represent the population.

Directions:

- 1. In this activity, participants examine the question "What is the average number of years of teaching experience for participants?" The activity requires that the participants develop three different sample representations of the data: 1) judgment sample, 2) convenience sample, and 3) random sample.
- 2. Create several judgment samples. Ask the participants to make a prediction about the average number of years of teaching experience. To create this sample, have them ask five participants who they deem representative of the population in the room. Have the participants put their average, rounded to the whole number, on a sticky note and build a "sticky note line plot" on the board. To build the line plot, draw a horizontal line on the board and mark a scale on it in whole number units. Have participants put the sticky notes on the board above the scale mark that goes along with their average, stacking repeat numbers. Ask the participants to comment on what they see with these sample averages, prompting them with the following questions:
 - Did everyone get the same sample average?
 - How much variation is there in the averages?
 - What would your prediction of the population average be?
 - Do you feel very comfortable with your prediction? Why?



- 3. Create several convenience samples. Ask the participants to take a convenience sample of the population by selecting five participants near them either sitting at their table or close by. Have the participants put their average on a sticky note and build a "sticky note line plot" on the board. Again ask the participants to comment on what they see with these sample averages. Ask the following questions:
 - Did everyone get the same sample average?
 - How much variation is there in the averages?
 - What would your prediction of the population average be?
 - Do you feel very comfortable with your prediction? Why?
- 4. Compare the two line-plots created in section 2 and 3.
- 5. Create several random samples. Collect the data from the participants in the room. As you do this, have one person at each table collect the data on the data sheet. Have the participants cut out the data into small squares and place them into a bag or basket. Have each participant take a sample of five squares from the data. The squares should be replaced before another participant takes a sample. Have the participants calculate the average for their sample and create a third "sticky note line plot." Examine the line plot, determining its mean, and comparing it to the mean and variation of the other two line plots. Ask participants to comment on the differences in predictions one would make from the three types of samples. Overall, we should see that the distribution of sample averages from the random sample will be less varied than the other samples and should be an unbiased representation of the data.
- 6. Find the true average for the population of participants in the room. Compare this average to the averages developed from each of the sampling methods. If all works well, the random samples overall should be the most accurate. However, samples will vary and therefore, some random samples will not be as good as some of the judgment samples. In addition, the samples are small; therefore more variation should be expected. It is possible that the random sample does not provide the most accurate estimate of the true average. If there is time to discuss the concept of sample size, it would be helpful to repeat the random sampling process but have the participants take samples of ten rather than five squares and collect the average data once again.



Data Recording Sheet

	<u> </u>	



Activity: Household Data

Format: Pairs/Small group

Objectives: Participants will organize a set of data using one of the data

organizers discussed in the session. Pairs will share what their

data looks like using the appropriate organizing tool.

Related SOL: K.15, 1.18, 2.23, 3.21, 4.20, 5.18, 6.18, 7.18, 8.13

Materials: Chart paper, markers/pens, tape, Household Survey Data

Activity Sheet

Prepare index cards beforehand with the following data organizer directions (one direction per card):

1. Use a list to show the number of adults in each household.

2. Use a chart to show the ratio of cars to adults.

- 3. Use a chart to show the ratio of TVs to people.
- 4. Use a frequency table to show the number of people in households.
- 5. Use a stem-and-leaf plot to show the number of TVs in households.
- 6. Use a box-and-whisker plot to show the number of children in households.
- 7. Use a frequency table to show the number of cars in households.
- 8. Use ordered pairs to show the relationship between the children in a household and the total number of people in a household.
- 9. Use a matrix to show the number of children versus the number of cars per household.
- 10. Using a tree diagram, show the total number of combinations of children and cars.

Time Required: 30 minutes

Directions:

- 1. Organize participants into pairs. Explain to the participants that they will be working together to organize a set of data.
- 2. Brainstorm together ways that data can be organized. Make a chart or write ideas down on a transparency.
- 3. When all methods are thought of/discussed, give each pair an index card with a data organizer written on it. Give all pairs the same Household Survey Data Activity Sheet.



- 4. Pairs need to use the data organizer from their index card to organize the Household Survey data they just received onto chart paper.
- 5. Have pairs post their displays on the wall for others to see.
- 6. If time, have pairs share the similarities/differences they notice about the different ways the data is displayed.
- 7. Discuss why the same set of data looks different. Does the organization of the data change its meaning?



1999 Survey of Household Members

1999 Survey of Household Members						
Name	Children in Household	Total Number of People in Household	Number of TVs in Household	Number of Cars in Household		
Adams	2	4	4	2		
Brown	1	3	2	2		
Bury	0	3	3	3		
Chambers	4	6	5	3		
Cleveland	1	3	2	2		
Critzen	2	3	3	1		
Cunningham	2	4	2	1		
Davis	3	5	4	3		
Dumante	1	2	2	2		
Elliot	2	3	3	3		
Gale	0	1	2	1		
Galland	0	2	3	2		
Herrig	2	4	4	2		
Kerby	1	3	2	2		
Kincaid	2	4	2	2		
Leigh	2	3	3	1		
Lowe	1	3	2	2		
Mazick	2	4	3	2		
Martinez	3	5	5	3		
Moore	2	3	4	2		
Nunez	2	5	4	3		
Pranter	2	4	4	1		
Richards	0	2	1	1		
Riley	0	1	3	1		
Roberts	1	3	3	2		
Shaw	2	3	3	1		
Whitten	1	2	2	1		



Activity: Grab a Handful

Format: Large Group

Objective: Participants will collect data and use the data to illustrate

measures of central tendency and range.

Related SOL: 5.19, 6.19, 7.16

Materials: Container of linking cubes, sticky notes

Time Required: 30 minutes

Directions

1. Participants grab a handful of cubes from a bucket or bag. The handful should be as large as possible. (At this point the leader should take only two cubes to be used in the discussion of *outliers* in #5.)

- 2. Each participant connects all of his cubes and writes the number of cubes in his train on a sticky note.
- 3. Participants then take their sticky notes to the chalkboard and construct a bar graph or a line plot.
- 4. Participants then line up at the front of the room according to the number of cubes each participant has. They should line up in front of each other when there is more than one participant with the same number of cubes. This human representation of the data should be similar to the line plot or bar graph on the board with sticky notes.
- 5. The instructor leads the group through a discussion of the measures of central tendency using the vocabulary words. While discussing the range, participants learn about outliers. Explain that an outlier is a value widely separated from other values in the data set. The two cubes that the leader picked in #1 above represent an outlier, as this value should be far separated from the value of the smallest handful.
- 6. Before discussing the words mode, median and mean, participants should be asked what they know. Assessing prior knowledge is key. They also should be encouraged to predict the median and the mean. The mode is easily illustrated on the human representation of the data and the line plot on the board.
- Before the discussion of the median begins, ask participants to spread out in one line, maintaining the sequential order of the data from least to greatest.
- 8. Ask participants to move out from the line in pairs one from each end at the same time. This allows participants to see how, as you approach the



- center, one or two participants will remain. They experience the meaning of the center. The movement away from the center makes this very clear to participants. Discussion should follow regarding the median and what it represents.
- 9. Before illustrating the mean, participants should discuss again what they are really trying to find. Encourage participants to share ways they could find the mean using the cubes.
- 10. Have participants share linking cubes with each other trying for all participants to have the same number of cubes. They share until sharing anymore isn't helpful to reach the same number of cubes for each participant. More than likely two groups will exist. Some participants will have trains of one number and the other group will have trains of another number. At this time participants discuss the mean and also use the term average.
- 11. End the session by reviewing the measures of central tendency and stating again the actual answers for this collection of data. Participants should be encouraged to discuss how changes in the data would affect the measures of central tendency. Participants should explore the results of changing the data near the center as well as at the extremes. Have participants compare their results and describe the changes that occur. Ask participants to complete the Recording Sheet.



Grab a Handful

Recording Sheet

Name Date
Number of cubes I grabbed in one handful
Number of participants participating in this activity
Record the number of cubes collected by each participant on the back of this sheet.
Using the data, I discovered the following:
Range of the data
Identify any outliers, and tell why they are outliers.
Mode(s) of the data
How did you know?
Median of the data
What is the meaning of median?
Mean of the data
Why is the mean also called the average?
Which measure of central tendency best represents the data and why?



Activity: What's Missing?

Format: Whole Group

Objectives: Participants will learn the different parts of a graph and the

importance of each part.

Related SOL: K.15, 1.19, 2.23, 3.22, 4.20, 5.18, 6.18, 7.18, 8.12

Materials: Activity Sheets of graphs of survey results about candy, Candy

Graph A, B, and C

Time Required: 10 minutes

Background: As we begin to teach data representation methods to children,

it is important to discuss the "big picture" of the data. What is this data representing? How do you know? What information on the graph tells you what you need to know? What is the

sample size? What is the scale?

Oftentimes, children don't understand that each unit on the graph is of equal size and, therefore, represents an equal amount. It is the number of units that shows the amount one is talking about. A *scale* is a system of marks in a given order and at specific intervals. The interval should be constant. In most graphs, each *axis*, or reference line, should be labeled to show the categories being represented as well as the scale. If a picture or symbol is being used, a *key* should be included which states the unit value of the picture or symbol. The *title* should indicate the information the graph represents.

Sometimes a **sample size**, or number of people surveyed, is included, but can also be determined from some graphs by

totaling the number of objects represented.

Directions:

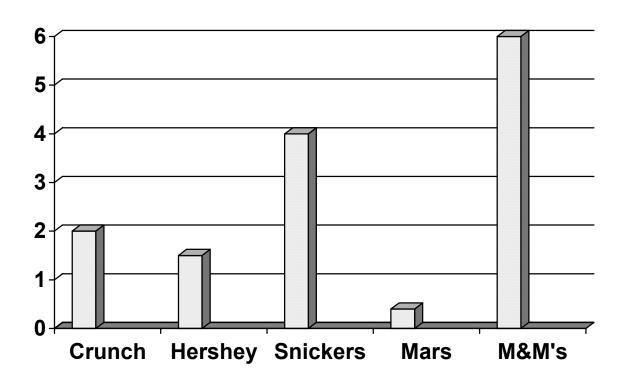
- 1. Display Candy Graph A.
- 2. Ask participants what they think this information represents. What would be a good title for this graph? How would you label the axes? Write down suggestions.
- 3. Display Candy Graph B.
- 4. Repeat the above questions. What is different about this graph? How does this new piece of information change the way one interprets this graph?



5. Display Candy Graph C and discuss all parts of the graph including labels, axes, title, scale, etc.

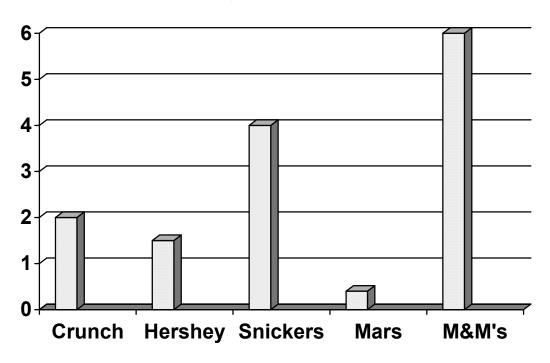


Candy Graph A





Candy Graph B

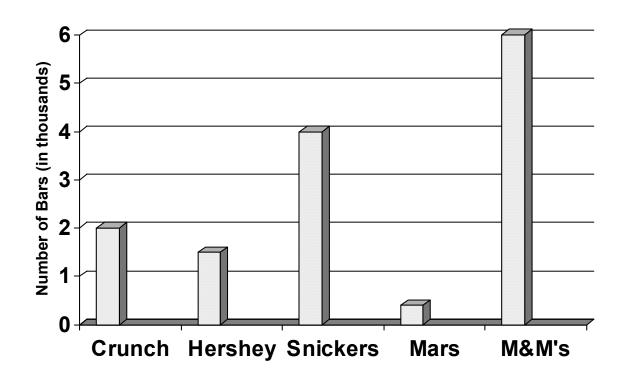


Data is in thousands



Candy Graph C

Number of Candy Bars Sold in the Cafeteria in 1999



Type of candy bar



Activity: Object Graphs and Picture Graphs

Format: Whole Group

Objectives: Participants will learn how to construct a pictograph and a bar

graph.

Related SOL: K.15, 1.19, 2.23, 3.22, 4.20, 5.18, 6.18

Materials: Multilink or unifix cubes of various colors, construction paper

squares to match the colors of the cubes, t-shirt pattern,

scissors, crayons or markers, grid paper

Time Required: 30 minutes

Background: An **object graph** uses a three dimensional object to represent

a unit.

A **picture graph** uses a picture or symbol to represent an object. If there is more than one of the objects, multiple representations of the symbol are used. A key should be

included that states the value of the symbol.

Number of teachers

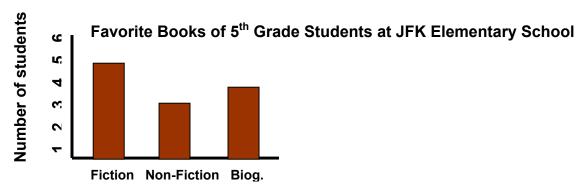
Teachers' Favorite Colors



Blue Green Red



A **bar graph** differs from a pictograph in that either horizontal or vertical parallel bars are used to represent counts of data in several categories. A bar is used for each category with the length of the bar representing the total count for that category.



Directions:

Object Graph

- 1. Place a handful or two of colored cubes on each table.
- 2. Ask participants to take a cube that most closely matches the color of their clothing.
- 3. Collect the data by having participants place their cubes on the matching colored paper square that the instructor has placed around the room.
- 4. Connect all of the cubes on the colored squares and make an object graph with the cubes. Place them on the floor or tape them on the wall.
- 5. Add a title, label the axes, and create a key.

Picture Graph

- 1. Next, have participants color in the t-shirt pattern to match their cube and cut it out.
- 2. Ask for suggestions on how to organize the data (sort by colors).
- 3. Tape each t-shirt up on the wall to construct a pictograph with appropriate labels and key.
- 4. Ask participants for the differences they notice between the two graphs constructed so far.

Bar Graph

- 1. Give each participant a piece of grid paper.
- 2. Have them construct a bar graph from the t-shirt color data.
- 3. Discuss differences between the three types of graphs.
- 4. As a large group, discuss other ideas for data collection appropriate in a classroom. Distribute copies of "Ideas for Graphing."



<u>Ideas for Graphing</u> (adapted from Marcy Cook)

Bar Graphs

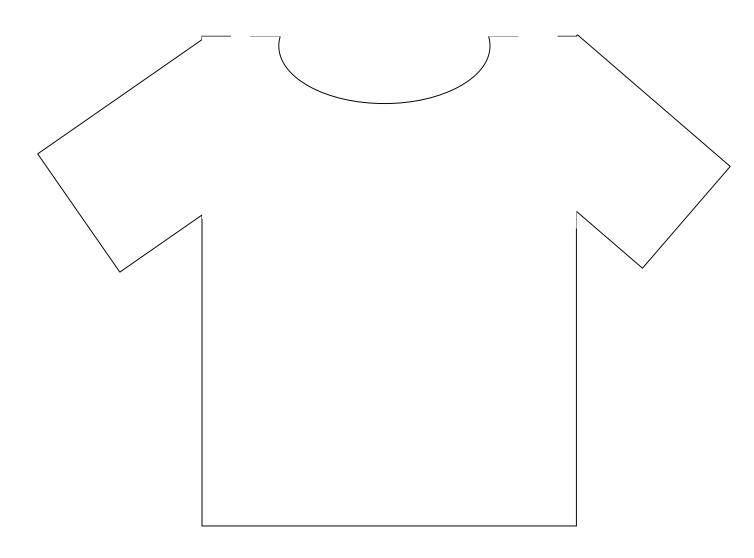
- In which quarter of the year is your birthday?
- Is the date of your birthday even or odd?
- Estimate how many times you went swimming last summer.
- What is the sum of the digits in your address?
- What is the difference between the largest digit in your address and the smallest digit in your address?
- How many objects in your desk have a number on them?
- How many doorways can you walk through in your house?
- What is the sum of the digits in your telephone number?
- How many times do your eyes blink in one minute?
- What is your favorite subject in school?
- What subject do you think we should spend the most time on?
- Write a complete sentence. Which letter did you use the most?
- How many times can you write the number 8 in one minute?
- How many different words ending with the letter "E" can you write in one minute?
- Approximately how many times do you get out of your seat on a school day?
- What time do you wake up on a school day (to the nearest quarter hour)?
- How many different vowels are used when you write your full name?
- Determine the fractional part of your name that is consonants.
- Collection of Halloween candy.

Line Graph

- Keep track of your bedtime for one week.
- Keep track of how many glasses of water you drink for one week.
- Collect the daily temperature for one month.
- Attendance (class or school) for a week, month, etc.
- Stock prices for a week, month, etc.
- Money collected each hour during a school fundraiser (car wash, bake sale, etc.).
- Number of visitors to your house every ten minutes on Halloween.
- Amount of money collected by the lunchroom each day for one week.
- Number of math problems assigned for homework each day for one week, month, etc.
- Length of time it takes you to get to school each day for one week.
- Amount of time you spend on homework each night for one week, month, etc.



T-shirt pattern





Activity: Attributes of Bar Graphs and Attributes of Polygons

Format: Large Group/Small Group

Objective: Participants discover the attributes of well-constructed bar

graphs.

Related SOLs: 2.23, 3.21, 3.22, 4.20, 5.18, 6.18, 8.12

Materials: Collection of Polygons Activity Sheet, paper, rulers, colored

markers

Time Required: 30 minutes

Directions:

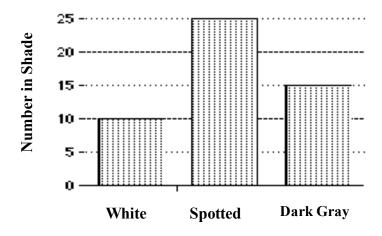
1. The participants examine the 50 polygons on the Collection of Polygons Activity Sheet and, working in pairs, choose an attribute that they can use to categorize the polygons.

- Participants cut apart and place the polygons in sets based on the attribute that they chose. All polygons must belong to one category according to the attribute.
- The instructor describes and illustrates the characteristics of a wellconstructed bar graph.
 - A bar graph is a graph of data in which parallel bars are used for comparing information from several categories. Each bar represents one category and the length of the bar represents the number of times that category occurs.
 - The categories are represented on one axis of the graph by the bases of the bars touching the axis. Each of the bars, extending from the axis, represents a category and each bar is the same width. Equal empty spaces are left between bars. An equal empty space is left before the first bar and after the last bar.
 - The number count of how many are in a category is represented by a scale on the other axis. This axis is marked off beginning with a number lower than the lowest count of items in any category, not necessarily zero, and continuing to or above the highest count. Lines may extend from this axis behind the bars to make them easier to read.
 - The category axis should be labeled to describe the categories being counted and the numerical count axis should be labeled telling what the numbers represents.
 - The graph should have a descriptive title.
 - The instructor's example has all the attributes described and shown in the transparency.



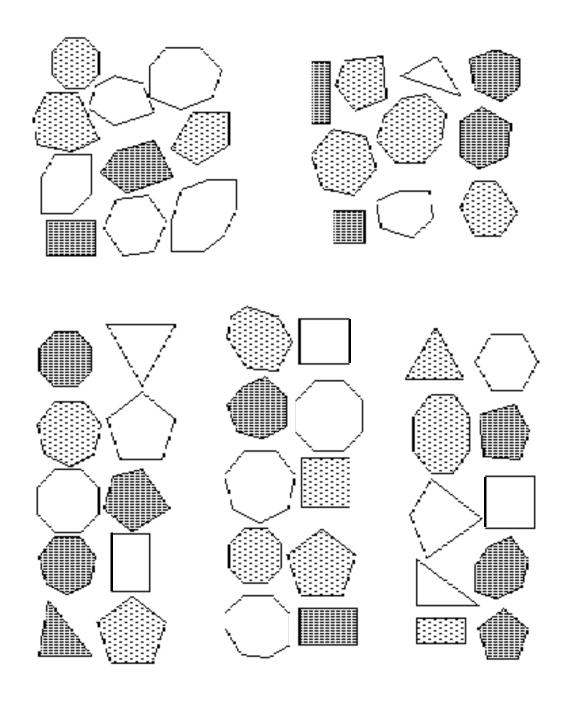
- 4. The participants work in pairs to construct a bar graph using the attribute that they chose.
- 5. When the pairs of participants have completed their graphs, they share them with other participants and assess whether or not they have included all the attributes of a well-constructed bar graph.
- 6. Each pair of participants chooses a different attribute of the set of polygons and constructs another bar graph based on the newly-chosen attribute.
- 7. When the pairs of participants have completed their second bar graph, they share them with another pair of participants and assess how well their graphs meet the attributes of a well-constructed bar graph.
- 8. Each participant chooses another attribute and constructs a bar graph independently. The bar graph should include all the attributes of a well-constructed bar graph. Below is one example of a graph participants may construct:

Shading of Polygons



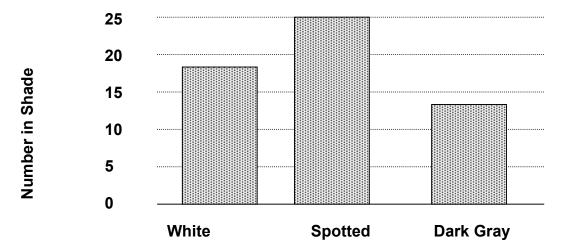


Collection of Polygons





Shading of Polygons





REPRESENTING DATA Session 3

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Representing Data	Line Plots	67	7.17, 7.18	Vocabulary Terms, Line Plots, Construction of Line Plots, What's In Your Box of Raisins?	Transparency, markers, one box of raisins per participant
	Stem-and- Leaf Plots	73	5.18, 6.18, 7.17	Age of Teachers' Children, Data from Math Classes	Transparencies, overhead transparency markers, sticky notes in two colors, chart paper, markers
	Box-and- Whisker Plots	79	5.18, 6.18, 7.17, 8.12	Vocabulary, Assessing Box and-Whisker Plots, Vocabulary Signs	Vocabulary terms on word cards, overhead markers, 3 x 5 index cards, yarn, scissors, digital camera
	Ham and Cheese Please (Coordinate Graphs)	88	4.18, 7.12		Two colors of tape, coordinate cards
	Find the Mole Hole (Coordinate Graphs)	89	4.18, 7.12	Mole Hole Grid	
	Graph Paper Warfare (Coordinate Graphs)	91	4.18, 7.12	Graph Paper Warfare Grid	
	Scattergrams	93	7.17, 8.12	Recording Sheet, Idea Sheet	Measuring tapes, graph paper
	Graph Detective	97	1.19, 2.23, 3.22, 4.20, 5.18, 6.18, 7.18, 8.12	Would You Draw the Same Conclusion?	



Activity: Line Plots

Format: Large Group; Pairs

Objectives: Participants will construct and compare line plots, and discuss

data distribution using these plots.

Related SOL: 3.21, 7.17, 7.18

Materials: Blank transparencies, overhead pens, blank paper, Vocabulary

Terms Activity Sheet, Line Plots Activity Sheet, Constructing Line Plots Activity Sheet, What's In Your Box of Raisins? Activity Sheet, small box of raisins for each participant

Time Required: 30 minutes

Background: A line plot is a graph that shows each item of information on a

number line. It has the shape of a bar graph. A line plot is often used to show the spread of the data. You can quickly identify the range, the mode, and any outliers of the data.

Graphs that show data distribution (data groupings) are usually called plots. This is because you show individual data points and not bars or connected lines. These graphs can be used to illustrate how some events are related to other events (age to height) or whether data tend to bunch together or spread out. Words used to describe data distributions are listed and defined in the following vocabulary section.

U-shaped – data that have large clusters of points at both ends and few points in the middle

Normal – data that assume the shape of a bell curve with few data points at either end building to many points at the center

Skewed – Data can either be **right skewed** by beginning with few data points and gradually rising to many data points or **left skewed** by beginning with many data points and gradually descending to few data points.

Bi-modal - Data that contain two distinct modes of equal height. Data containing three distinct modes are called **tri-modal**.



Directions:

- 1. Distribute Construction of Line Plots Activity Sheet to participants. Construct a line plot of the number of letters in the first name of each participant on the blank transparency. Briefly discuss how to construct a line plot (Construction of Line Plots Activity Sheet). Ask participants to discuss what they notice about the data (the range, median, and mode(s) of the data).
- 2. Using the line plot on the transparency, discuss the data display. Use the shape of the plot to introduce the idea of data distribution asking such questions as "What shape does the line plot have?"
- Discuss the data distribution vocabulary on the Activity Sheet. Discuss the meaning of the terms and sketch the shape each might take. Use the Sample Line Plots Activity Sheet to display sample data distribution of line plots.
- 4. Give each participant a box of raisins. Have them predict the number of raisins in the box. Do a line plot of their predictions on the What's In Your Box of Raisins Activity Sheet. In this case, the instructor must first ascertain the range of the data and then plot the points. Discuss outliers if a participant predicted a number significantly greater than other participants. Next, ask the participants to open their boxes and count the raisins. Have the participants construct a line plot on their What's in Your Box of Raisins Activity Sheet as each participant shares the number of raisins in his/her box. As the participants construct their line plots, construct the same plot using the Activity Sheet.
- 5. Conclude by discussing the distribution using the previously defined vocabulary. Have the group brainstorm the kinds of data that might show the different data distributions (e.g., average daily temperature in Atlanta from August to July should show a U-shaped distribution).



Vocabulary Terms Line Plots

U-shaped-

Normal-

Skewed-

Bi-modal-

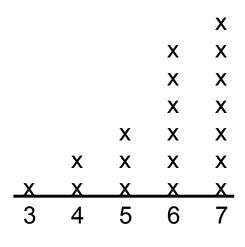


Line Plots

		Χ		
		Χ		
	X	Χ	Χ	
	X	X	Χ	
X	Χ	Χ	Χ	Χ
3	4	5	6	7

normal

U-shaped



right skewed

tri-modal



Constructing Line Plots

A line plot is a graph that displays the location of the data points along the segments of a number line. There is a one-to-one correspondence between the number of points on the graph and the number of collected data points. For any given value, the number of data points is represented by the number of "marks" written above the numbered segment on the number line.

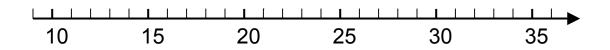
To develop a line plot, follow these steps.

- Step 1 Find the largest and smallest values in the data set.
- Step 2 Draw a number line that includes these values. Be sure the number line extends past the smallest and largest values.
- Step 3 Label the number line with a scale. Place a "mark" (usually an x or a dot) on the real number line that represents each data point.
- Step 4 Title the graph.
- Step 5 Look for the patterns in the data and make statements about the data set. Consider things such as the shape of the data (the distributions), the median, range, modality, etc.



What's In Your Box of Raisins?

Line Plot





Activity: Stem-and-Leaf Plot

Format: Large group; Pairs

Objectives: Following a discussion of the stem-and-leaf plot, participants

will work in pairs to collect data. The data will then be graphed by the whole group. Participants will again work in pairs to construct back-to-back stem-and-leaf plots comparing test data

from two classes.

Related SOL: 5.18, 6.18, 7.17

Materials: Blank transparencies (5 or more), overhead transparency pens,

small post-it notes in two colors, Background Information Activity Sheet, Copies of Test Data Activity Sheet, Test Data Activity Sheet, Stem-and-Leaf Plot Activity Sheet, chart paper,

magic markers, clock with second hand (optional)

Time Required: 30 minutes

Background: A stem-and-leaf plot is a useful way to display data that range

over several tens (or hundreds). The stem represents the tens

and the leaves represent the ones. Each number is

represented by one stem and one leaf.

Students surveyed fifth grade teachers at Ames School to find out the ages of their teacher's sons and daughters. The results of this survey are displayed on the stem-and-leaf plot below.

Ages of Fif	<u>th Grade Teachers' Children</u>
0	2
1	0, 4, 5, 5
2	0, 4, 5, 5 3, 6
3	9

key: 2 3 = one child age 23

As shown in the plot, the fifth grade teachers have a total of eight children ranging in age from 2 to 39 years. The median (15) and mode(s) (15) of the data are displayed on the stemand-leaf plot and can easily be determined. Clusters can be identified; for instance, more teachers have teenagers than toddlers.



Two sets of comparable data can be displayed on a back-to-back stem-and-leaf plot.

Ages of 6 th Grade Teachers' Children		Ages of 5 th Grade Teachers' Children
9, 7, 6, 5, 4, 3, 1	0	2
6, 4, 2	1	0, 4, 5, 5
	2	3, 6
	3	9

Students can compare the information presented; find the range, mean, median, and mode; locate clusters; and make inferences such as the fact that 6th grade teachers' children are younger than the fifth grade teachers' children.

Directions:

- 1. Use a transparency of "Ages of Teachers' Children" to go over background data with the whole group. Be certain to include:
 - Each number is represented by a combined stem and leaf. Each leaf may contain only one digit, but a stem may contain more than one digit For example, 123 would be represented by a stem of 12 and a leaf of 3.
 - The leaves are arranged from the stem outward and are in numerical order.
- 2. Have the participants pair up to collect data on how long each can hold his or her breath. Give each participant one color-coded post-it note. (The colors will be used for demonstrating an easy way to divide data into two groups. For example, you can give males one color and females another or you can give fifth and sixth grade teachers one color and seventh and eighth grade teachers another.) Have each participant time how long their partner can hold his or her breath, recording that number on the appropriate post-it note.
- 3. Collect the post-it notes and arrange them all by decades. Using a blank transparency, construct a stem-and-leaf plot using the data collected. Follow the model in the Background Information.
- 4. Briefly, have the participants discuss with their partner what they see on the plot. Have them share their ideas. Be certain to discuss:
 - All the data is visible on a stem-and-leaf plot.
 - It is easy to find the range and mode(s) of the data just by looking. This would be an appropriate time to introduce the terms "bimodal"



- (having two modes) and "trimodal" (having three modes) as it often happens in this type of data collection.
- Share finding the median of the data by counting. Remind the participants that, when starting with the largest number, one must count backward to find the median.
- The mean can be found in the normal manner.
- Discuss any other interesting clusters or trends that the group sees.
- 5. Discuss using two sets of comparable data to construct back-to-back stemand-leaf plots. Dividing the data into two sets by using the color-coded sticky notes makes this an easy task. Use another blank transparency and five or six pieces of data from the two colors of notes to quickly construct a sample of a back-to-back plot showing the participants how to collect the information on one stem-and-leaf plot and then reorganizing it on another.

Example:	record		reorganize		
	stem	leaf	stem	leaf	
		5,3,7		3,5,7	
	2	1,8,4,7	2	1,4,7,8	

- 6. Give each pair of participants a copy of the grade data from two math classes. Have them construct a back-to-back stem-and-leaf plot from the data. Have them briefly analyze what they found prior to a whole group discussion.
- 7. Use a transparency of the grade data to share the measures of central tendencies for each class. Have the participants analyze the data discussing such things as why one class may have done better than the other.
- 8. Have the whole group brainstorm suggestions for question stems that could be represented on stem-and-leaf plots such as hopping on 1 foot for 30 seconds and using data from other sources.



Ages of Teachers' Children

Data Set for 5th Grade Teachers:

2, 10, 14, 15, 15, 23, 26, 39

Stem-and-Leaf Plot

Ages of Fifth Grade Teachers' Children

key: $2 \mid 3 =$ one child age 23

Back-to-Back Stem-and-Leaf Plots

Ages of 6 th Grade Teachers' Children		Ages of 5 th Grade Teachers' Children
9, 7, 6, 5, 4, 3, 1 6, 4, 2	0 1 2 3	2 0, 4, 5, 5 3, 6 9



The following are scores obtained by two classes of 25 grade five students on a math test. Compare the two sets of scores by using back-to-back stem-and-leaf plots. What conclusions might you draw by studying the data displayed in this way?

Class A	96	75 54 88	80	75	69	 	 	
Class B	75	80 76 69	79	80	87	 	 	



Data from Math Classes

The following are scores obtained by two classes of 25 fifth grade students on a math test. Compare the two sets of scores by using back-to-back stem-and-leaf plots. What conclusions might you draw by studying the data displayed in this way?

Class A	73 96 69	75 54 88	42 80 80	93 75 52	69	62 66	62 81	37 79	73 83	76 56
Class B	65 75	80 76	67	80	87 97	44	82 54	71 56	91 57	93
	75 62	76 69	79 75	80 80	87 91	83	34	30	31	82
7, 7, 3,	•	7, 9, 7, , 6, 5, , 0, 0,	•		3 4 5 6 7 8 9	7 2 2, 2, 3, 0,	4, 6, 2, 6, 3, 5, 0, 1, 6	9, 9 5, 6,		
range - median - mode -	49 79 80					med	ge - dian · de -	62 ,	•	0, 88
mean -	74.6	64				mea	an -	70.7	•	0 , 00



Activity: Box-and-Whisker Plots

Format: Whole Group/Small Groups/Pairs

Description: Following a brief discussion of the term median, the vocabulary

listed below, and box-and-whisker plots, data will be gathered from the whole group. A human box-and-whisker plot will be constructed from the data. Small groups will then work

constructed from the data. Small groups will then work together to compare two box-and-whisker plots as a sample

assessment item.

Objectives: Participants will have a model for showing students what a box-

and-whisker plot looks like and how to construct it. Participants will be able to construct a box-and-whisker plot using a set of

data.

Related SOL: 5.18, 6.18, 7.17, 8.12

Materials: Large picture of a cat or tiger (optional), vocabulary terms on

word cards, overhead marking pens, 3 x 5 cards (one per participant), thick craft yarn, scissors, 6 signs with string to hang around the necks of students. The signs will have the following labels: median, lower extreme, upper extreme, lower quartile, upper quartile, and interquartile range (masters are included). Line on the ground (tape on the floor, chalk on concrete, line in the tile, etc.), Polaroid camera or video camera (optional), blank transparency, copies of worksheet - Assessing

Box-and-Whisker Plots (1 per participant)

Time Required: 45 minutes

Background: A box-and-whisker plot is a type of graph used to represent

data. It is most appropriate when you want to show the

median, first and third quartiles, and least and greatest of a set of data. The "box" is like a cat's face and the "whiskers" are formed by the data that extend out from the box. There is specialized vocabulary when using box-and-whisker plots.

Vocabulary: Median: the middle item in a set of data listed in numerical

order; for an even number of items, the median is the average

of the two middle items

Lower Extreme: the smallest numerical piece of data in a set



Upper Extreme: the largest numerical piece of data in a set

Lower Quartile: the median of the lower half of the data; the point that separates the first and second quartiles

Upper Quartile: the median of the upper half of the data; the point that separates the third and fourth quartiles

Interquartile Range: the data included in the second and third quartiles; the data encompassed within the "box"

Directions:

- Go over the vocabulary for this topic using a transparency of the vocabulary and writing in the definitions. If you have a picture of a cat or tiger, briefly show how the face forms a box with the whiskers coming out the side.
- 2. Set the context by telling the participants we want to find the "average" number of letters in the first and last names of the participants in the group in order to decide what to charge for class t-shirts. The manufacturer charges per letter to personalize them, but we want to charge everyone the same rate so we will look at a way to find that rate by using a box-and-whisker plot. Have each participant write the sum of the letters in their first and last name on the 3 x 5-index card.
- 3. Have the participants stand and organize themselves in a line (using the line on the floor) from the least sum to the greatest. Check for proper placement and discuss any discrepancies. Have the students hold their cards in front of them.
- Locate the pertinent points in this data set.
 - Locate the **lower extreme** (the smallest sum) and hang that sign around the neck of the person representing that point. Repeat by hanging the **upper extreme** sign around the neck of the person with the largest sum.
 - Now locate the median by having the lower extreme wave to the upper extreme. Continue to move to the center of the line by having each successive person from both ends of the line wave to each other. The last person to wave should be the median with an equal number of participants on both sides. (NOTE: If two students are in the middle, you need to discuss the fact that the median would be the average of these two numbers. For a first experience, it is better to plan ahead and have an odd number of participants leaving only one person to be the median. The leftover person could be the designated photographer or verifier.) Hang the median sign around the neck of the person at that point.



- Find the median of the upper half of the data by repeating the waving process, this time having the wave start with the median and the upper extreme. Label this person as the **upper quartile** and discuss the fact that this person divides the upper half of the data in two equal pieces.
- Repeat this process to find the lower quartile using the median and the lower extreme. Hang the sign and discuss the fact that this person divides the lower half of the data in two equal pieces.
- Build the box to include the interquartile range using the thick craft yarn. Start with the end of the yarn at the lower quartile and have that participant hold the end of the yarn shoulder high. Run the yarn past the median (that participant may hold on to the yarn to stabilize it) and onto the upper quartile's shoulder. Drop the yarn down to the waist of the upper quartile and have it held there. Run the yarn past the median and back to the waist of the lower quartile, and then back up to the shoulder where you started. Cut the yarn. Thus, you have made the box. Have a participant on either side of the median hold the card that says interquartile range. Discuss the fact that the box encloses 50%, or half, of the group. It contains the middle half of all the data.
- To create the whiskers, tie a piece of yarn to the center of one side of the lower quartile end of the box and run it to the lower extreme. Cut the yarn and repeat this process from the upper quartile to the upper extreme.
- If a camera is available, take a picture of the final box-and-whisker plot so the participants can have the whole picture of what they made.
- 5. Use the blank transparency and the data collected to construct an accurate box-and-whisker plot. Use equal intervals for each sum along a line on the transparency. Have the participants help you reconstruct what was done to create the human box-and-whisker plot as you recreate the plot on the transparency.
- 6. Compare the human box plot with the one on the transparency. How do they compare? How do they differ? (Elicit that each person representing a piece of data had his or her own space in the human box plot. In the actual box plot, data with the same sum shared a point on a line, thus compacting the data. All the quartiles of the human box plot appear to be the same size, while on the actual plot their sizes may differ even though each quartile contains the same number of sums.)



- 7. With the whole group, brainstorm other scenarios for collecting data and constructing box-and-whisker plots. Some suggestions might include cost of CDs or some other item that students this age purchase or scores on a set of test papers. Add question stems to the charts from Session I Posing Questions.
- 8. Have participants form pairs and give each person a copy of Assessing Box-and-Whisker Plots Worksheet. Have them discuss the two box plots on the sheet. Then have each complete the paragraph as directed on the worksheet.
- 9. Reform small groups by combining two pairs. Have participants share their paragraphs in small groups and discuss how the paragraphs might be assessed.
- 10. Conclude the session with a summary and a short question and answer period.



Vocabulary

median –

lower extreme –

upper extreme -

lower quartile -

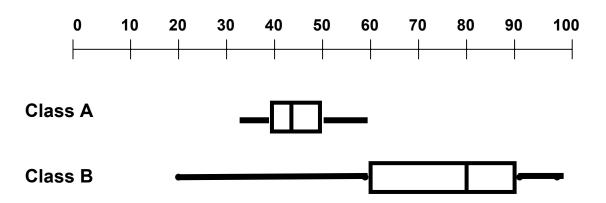
upper quartile -

interquartile range –



Assessing Box-and-Whisker Plots Worksheet

The following box-and-whisker plot represents the test scores for students in two different classes:



Write a paragraph comparing how these two classes did on this test. Give as much information as you can.



median

lower extreme

upper extreme



lower quartile

upper quartile



interquartile range



Activity: Ham and Cheese, Please!

Format: Pairs

Objectives: Participants locate points on a coordinate grid.

Related SOL: 4.18, 7.12

Materials: Tape, 4 x 6 index cards labeled with coordinates

Time Required: 10 minutes

Directions:

1. Lay out a coordinate grid on the floor using two different colors of tape – one for the horizontal lines and the other for the vertical lines.

- 2. Make a set of coordinate cards, enough for one card for every two participants. Each card has an ordered pair. Make a matching set of cards so that every point appears on two cards.
- 3. Divide the group in half one group is Ham and the other is Cheese. Give each Ham member a coordinate card from one set of cards. Remind them not to tell or show their coordinate to anyone. When directed, have the Hams move to the correct location on the floor grid.
- 4. Give each Cheese member a coordinate card from the matching set. Have the Cheese members try to guess which Ham will be their partner. When directed, have the Cheese members find their matching Ham.
- Partners have now been established for other activities.



Activity: Find the Mole Hole

Format: Pairs

Objectives: Participants play a game to locate a hidden Mole Hole by

making educated guesses on a coordinate grid.

Related SOL: 4.18, 7.12

Materials: 10 x 10 coordinate grid - one per student per game, colored

pencils

Time Required: 10 minutes

Directions:

Explain these rules:

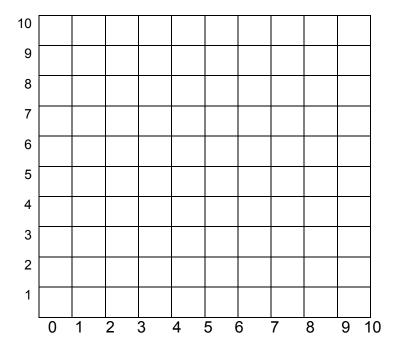
1. The goal is to find the Mole Hole in the least number of guesses.

- 2. One student is the MOLE and the other is the EXTERMINATOR. The MOLE makes a row of 5 points, horizontally, vertically, or diagonally, on his/her coordinate grid but does not show the secret row to the EXTERMINATOR. The secret row is the Mole Hole.
- 3. The EXTERMINATOR looks for the Mole Hole by naming points on the grid one at a time. The MOLE says "yes" or "no" to tell whether a named point is on the Mole Hole.
- 4. If a point is not on the Mole Hole, the EXTERMINATOR keeps guessing until he/she names a point on the Mole Hole. Once a Mole Hole point is found, the EXTERMINATOR has a clue to the location of the Mole Hole. The EXTERMINATOR uses strategy to name the other points that mark the entire Mole Hole.
- 5. The EXTERMINATOR uses his/her own grid to keep track of his/her guesses. This will help the EXTERMINATOR plan better guesses and avoid repeating points. Have pairs take turns as MOLE and EXTERMINATOR.
- 6. Winner is the partner who found the mole hole in the least number of guesses.



MOLE HOLE GRID

MOLE's GRID

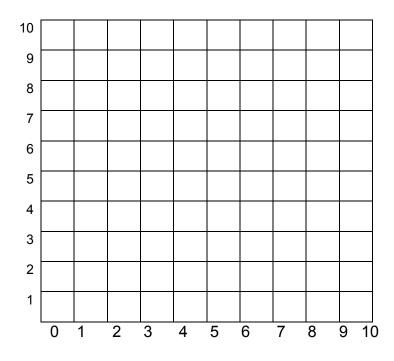


Coordinates called:

(,)	(,)	(,)
	(,)	
	(,)	
	(.)	

)

EXTERMINATOR's GRID



Coordinates called:

<u>_</u>	_,	_)	(_	,	_)	(_	,)
_	_,	_)	(_	,	_)	(_	,)
(_,	_)	(_	,	_)	(_	,)



Activity: Graph Paper Warfare

Format: Pairs

Objectives: Participants find points on a coordinate grid in preparation for

scattergrams.

Related SOL: 4.18, 7.12

Materials: Graph paper – 1 sheet per student per game, colored pencils or

markers

Time Required: 15 minutes

Directions:

1. Each player will locate 1 battleship, 2 destroyers, and 2 submarines on his/her grid. Ships may be placed horizontally, vertically, or diagonally. Ships may not overlap. Ships are defined as follows:

1 battleship – 6 consecutive points in a row;

1 destroyer – 4 consecutive points in row;

1 submarine – 3 consecutive points in a row.

(For younger players, fewer ships may be used and locations of ships restricted to Quadrant I)

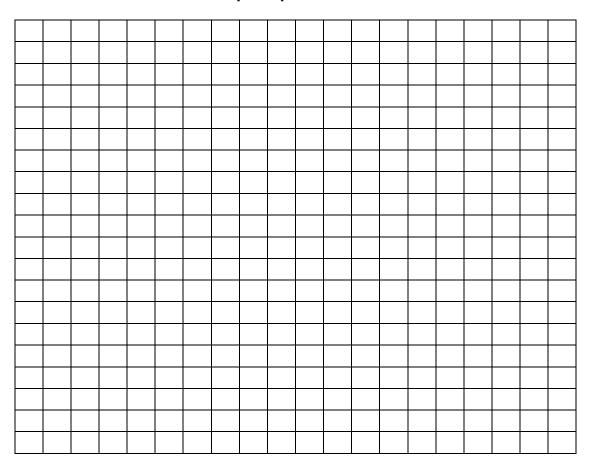
- 2. Player 1 (determined by a quick round of "Paper, Scissors, Rock") calls a coordinate believed to be a coordinate of part of a ship. Player 1 should record the coordinate called on his grid in a color different than the color of his own ships.
- 3. Player 2 responds with "Hit", if the given coordinate is part of a ship, and announces type of ship hit; or "Miss", if it is not a part of the ship.
- 4. If a Hit is scored, Player 1 calls a second coordinate and Player 2 again responds. If a Miss is scored, play goes over to Player 2 who calls a coordinate. Player 1 then responds with "Hit" and the type of ship struck, or "Miss".
- 5. Play continues in this fashion until an entire fleet is sunk.

NOTE: For a ship to be considered sunk, all coordinates containing the ship must be called.

Variations: Adjusting the size of the grid paper and/or the number or size of the ships can shorten or lengthen the game play.



Graph Paper Warfare Grid





Activity: Scattergrams

Format: Small group

Objective: Participants will use scattergrams to analyze relationships in

data.

Related SOL: 7.17, 8.12

Materials: Graph paper, measuring tape, recording sheet, graphing

calculator (optional)

Time Required: 30 minutes

<u>Background</u>: A scattergram, which is also called a scatter plot, suggests

whether or not two sets of data are related. The data are graphed as a collection of ordered pairs of numbers (x, y). Whenever you graph two sets of data as ordered pairs, you

make a scattergram.

To determine if the data in a scattergram are related, pretend that a line is drawn so that about half the points in the

scattergram are above the line and about half are below.

If the y-coordinates tend to increase as the x-coordinates increase, then x and y have a positive correlation. If the y-coordinates tend to decrease as the x-coordinates increase, then x and y have a negative correlation. If no pattern exists between the coordinates, then x and y have no correlation. In general, if the line between the points slants up and to the right, there is a positive relationship. If the line slants downward and to the right, there is a negative relationship. If no line is

apparent, there is no relationship.

Directions:

- 1. Pose the question: Do you think there is a relationship between a person's arm span and a person's height?
- 2. Each group should use a measuring tape to measure the height and arm span of each person in the group. Measurements may be in inches or centimeters. To measure arm span, lift both arms to shoulder height and measure from fingertip to fingertip across the back.
- 3. Record the value of each person's measurements in the chart on the worksheet.



- 4. Send a "runner" to each of the other groups in the room and record their data on the worksheet.
- 5. Graph the data on a coordinate plane. Label the horizontal axis "Height" and the vertical axis "Arm Span".
- 6. Draw a trend line. What kind of relationship does the trend line show? (As an extension, a graphing calculator could be used to determine the line of best fit for the data.)



RECORDING SHEET

NAME	HEIGHT	ARM SPAN
	<u> </u>	l .



Additional Ideas for Determining Positive, Negative, or No Correlations

- height and shoe size*
- salary and shoe size
- year and winning time at the Olympics
- student study time and test scores*
- age and value of a family car
- student's height and test scores
- height and age of a pine tree
- number of pets owned and your age
- number of hours of TV watching and test score
- number of trees in a small park and the temperature on a summer day
- weight and speed in a foot race
- test scores and shoe size*
- amount of education and annual salary*
- washing the car and rain fall
- outdoor temperature and layers of clothing
- eating nutritious food and being healthy
- speed of typing and the number of pages to be typed
- area of a soccer field and height of the grass
- age and height*
- age and value of antiques
- school attendance and grades*
- watching the news and scores on a current events guiz
- practicing basketball and the ability to play well
- taking a school bus and completing homework
- wrist and shoe size*

^{*}These topics might prove very interesting when the data are collected and graphed in a scattergram.



Activity: Graph Detective

Format: Pairs or small groups

Objectives: Participants will apply the skills they learned from the previous

activities to analyze graphs for missing attributes.

Related SOL: 1.19, 2.23, 3.22, 4.20, 5.18, 6.18, 7.18, 8.12

Materials: Set of graphs with missing information

Time Required: 20 minutes

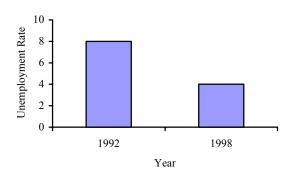
Directions:

1. To wrap up the session, give each group a set of three graphs and conclusions and ask them to discuss whether the conclusion is accurate and what factors about the graph may have lead to inaccurate conclusions. Key factors in the misleading graphs include the following:

- Graph 1: Missing years A trend cannot be seen by examining only two years. USA Today uses a lot of these graphs to compare two years. This graph compares unemployment in 1992 and 1998. Although 1998 may be lower than 1992, we should not assume that 1999 will be lower than 1998 or that the trend between 1992 and 1998 was downward. We need to see the other years to determine if there is a trend over time or if 1998 is a fluctuation.
- Graph 2: Broken scales Often graphs imply larger differences than are true because the scale is broken. In this graph, because the scale is broken, it appears that there were twice as many births in July than there were in June.
- Graph 3: Size Distortions Picture graphs using objects to demonstrate change can be misleading because the size of the objects may not truly represent the relative numerical value. This graph shows the relative earnings of men and women. Women earn approximately 70¢ for every dollar that men earn; yet, the graph implies that men earn nearly three times as much as women because of the relative size of the threedimensional bars.

Would You Draw the Same Conclusion?

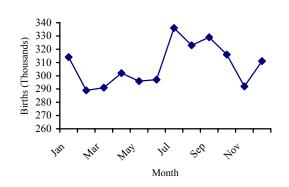
Unemployment Rates (1992 - 1998)



Conclusion: Unemployment rates have fallen steadily since 1992 and will continue to fall in the near future.

Group's Conclusion:

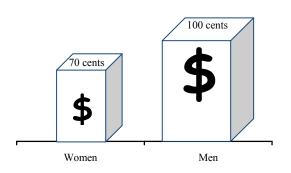
Number of Births per Month



Conclusion: The number of births doubled between June and July.

Group's Conclusion:

Earning Power of Men versus Women



Conclusion: Men make almost three times as much as women. The bar for men has nearly three times as much volume.

Group's Conclusion



MORE ON REPRESENTING DATA, ANALYZING DATA, AND INTERPRETING RESULTS Session 4

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Representing Data	Attributes of Circle Graphs	100	6.18, 8.12	Thirds, Fourths, Fifths, Sixths, Eighths, Mystery Graphs	
	Construct- ing Circle Graphs	107	6.18, 8.12	Favorite Ice Cream, Favorite Amusement Park Rides, Favorite Chocolate Treat	Protractors, compasses, rulers
	Frequency Distributions and Histograms	111	7.17, 8.12	Hand Full Sheet, First Histogram, Refined Histogram, Attribute Sheet	Bag of counters, rulers
	When It Rains	118	4.20, 5.18, 6.18, 8.12	When It Rains	
Analyzing the Data	Let the Graph Do the Talking	121	1.19, 2.23, 4.20, 5.18, 3.22, 6.18, 7.18, 8.12	Example Graph 1, Example Graph 2	
	Matching Game: Graphs, Data, Summary	125	1.19, 2.23, 3.22, 6.18, 7.18, 8.12	Matching Graphs, Data and Narratives (18 Sheets)	Large index cards on which to mount the data
	Name That Graph	144	4.20, 5.18, 6.18, 7.17, 8.12	Name That Graph graphs and Recording Sheet	
	Draw the Graph	147	1.19, 2.23, 3.22, 4.20, 5.18, 6.18, 7.18, 8.12	3 Graphs	
	Interpreting the Data to Inform the Question	151	All K-8 Statistics SOL	Sample Questions	



Activity: Attributes of Circle Graphs

Format: Large Group/Individual/Small Group

Objectives: Participants will analyze fraction, percent, and central angle

relationships in circle graphs.

Related SOL: 6.18, 8.12

Materials: Concept Activity Sheets of circle graphs (THIRDS, FOURTHS,

FIFTHS, SIXTHS, and EIGHTHS), each marked or to be marked and shaded (or colored) with a fractional part, a percent, and the measure of the central angle. Concept Understanding Assessment Activity Sheet: Mystery Circle

Graphs

Time Required: 10 minutes

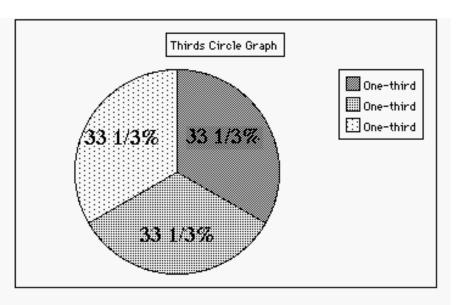
Directions:

 The participants are each given an activity sheet to complete, followed by the instructor's questions related to the shading completed (THIRDS, FOURTHS, FIFTHS).

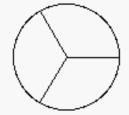
- 2. The participants shade a part of the interiors for SIXTHS and name the measures of the central angles. The instructor's questions about equivalents follow.
- 3. The participants shade a part of the interiors for EIGHTHS and name the percent equivalents and measures of the central angles. The instructor's questions about equivalents follow.
- 4. The participants use information from their THIRDS, FOURTHS, FIFTHS, SIXTHS, and EIGHTHS Activity Sheets to assist in completing the Mystery Circle Graphs activity.



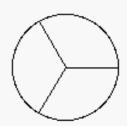
THIRDS



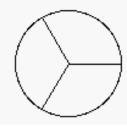
Shade 1/3 of the circle.



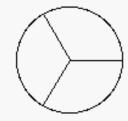
Shade 2/3 of the circle.



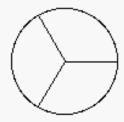
Shade 3/3 of the circle.



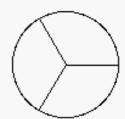
Shade 33 1/3% of the circle.



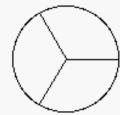
Shade 66 2/3% of the circle.



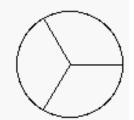
Shade 100% of the circle.



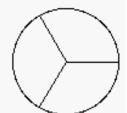
Shade a 120° angle.



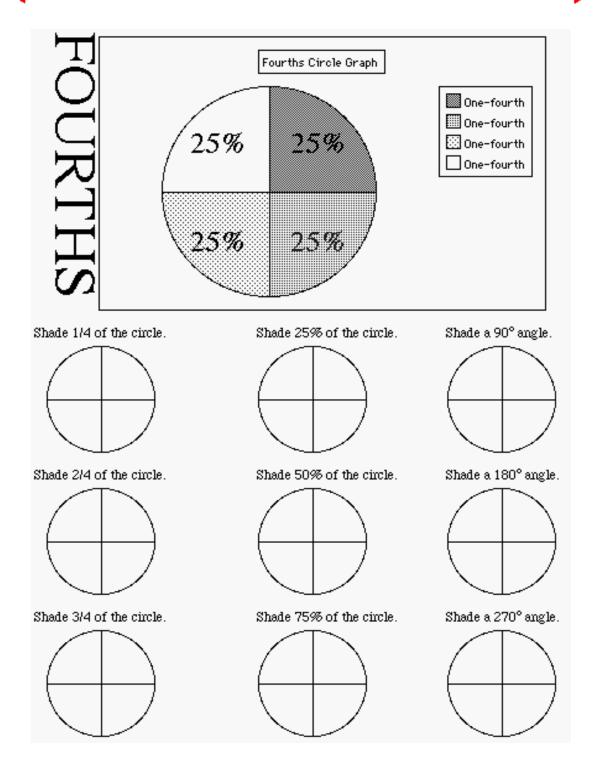
Shade a 240° angle.



Shade a 360° angle.



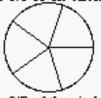




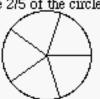


Fifths Circle Graph One-fifth One-fifth 20% 20% One-fifth One-fifth One-fifth 20% 20%

Shade 1/5 of the circle.



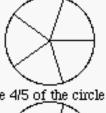
Shade 2/5 of the circle.



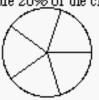
Shade 3/5 of the circle.



Shade 4/5 of the circle.



Shade 20% of the circle.



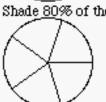
Shade 40% of the circle.



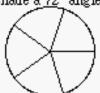
Shade 60% of the circle.



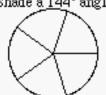
Shade 80% of the circle.

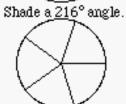


Shade a 72° angle.

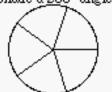


Shade a 144° angle.



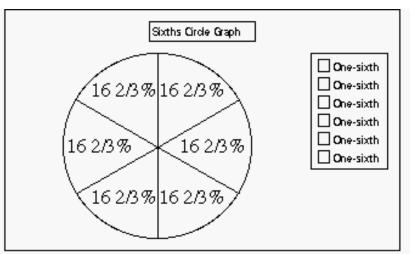


Shade a 288° angle.





SIXTHS



Shade 1/6 of the circle.



Shade 2/6 of the circle.



Shade 3/6 of the circle.



Shade 4/6 of the circle.



Shade 5/6 of the circle.



Shade 16 2/3%.



Shade 33 1/3%.



Shade 50%.



Shade 66 2/3%.



Shade 83 1/3%.



Shade a ____° angle.



Shade a ____° angle.



Shade a ____° angle.



Shade a ____° angle.

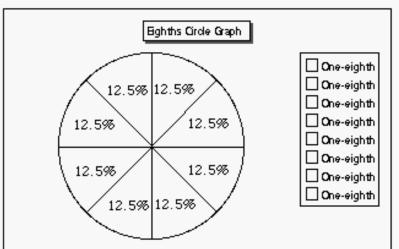


Shade a $__$ ° angle.





EIGHTHS



THS	12.5% 12.5%	☐ One-eighth ☐ One-eighth ☐ One-eighth ☐ One-eighth
Shade 1/8 of the circle.	Shade%.	Shade a° angle.
\bigcirc	\bigoplus	\bigcirc
Shade 2/8 of the circle.	Shade% .	Shade a° angle.
\bigoplus	\bigoplus	\oplus
Shade 3/8 of the circle.	Shade%.	Shade a° angle.
Shada 410 a faba atirala	Shade %5.	Shade a° angle.
Shade 4/8 of the circle.	Shade%.	Shade a angle.
\bigoplus	\bigoplus	\bigoplus
Shade 5/8 of the circle.	Shade% .	Shade a° angle.
\oplus	\bigoplus	\bigoplus
Shade 6/8 of the circle.	Shade	Shade a° angle.
	™ ***	
Shade 7/8 of the circle.	Shade%.	Shade a° angle.
AIX	KIX .	KIX .



Mystery Circle Graphs

For each sector in the circle graph, find the fractional part represented, the percent of the whole circle, and the measure of the central angle.

Mystery Circle Graph 1 Fraction You decide. Percent You decide. Central You decide. Angle Mystery Circle Graph 2 Fraction You decide. Percent You decide. Central You decide. Angle Mystery Circle Graph 3 Fraction You decide. Percent You decide. Central You decide. Angle Mystery Circle Graph 4 Fraction You decide. You decide. Percent You decide. Central Angle



Activity: Constructing Circle Graphs

Format: Large Group/Individual/Small Group

Objectives: Participants will analyze data by displaying it in circle graphs.

Related SOL: 6.18, 8.12

Materials: Compasses, rulers, protractors, construction of a circle graph

activity sheet (Favorite Amusement Park Rides), circle graph construction assessment activity sheet (Favorite Chocolate

Treat)

Time Required: 20 minutes

Directions:

1. The instructor describes the attributes of a circle graph and demonstrates how the sectors are determined.

A circle graph is a graph of data in which parts of a whole are represented as sectors of a circle.

Each sector, or pie-shaped wedge, usually contains the actual number or percent of the whole and a label of what the part represents. Some circle graphs use a legend to label the sectors of the graph. A sector is bound by two radii and an arc of the circle. An arc is part of a circle connecting two points on the circle. The whole is represented by the area of the circle. The parts are represented by the areas of sectors of the circle. The graph has a descriptive title.

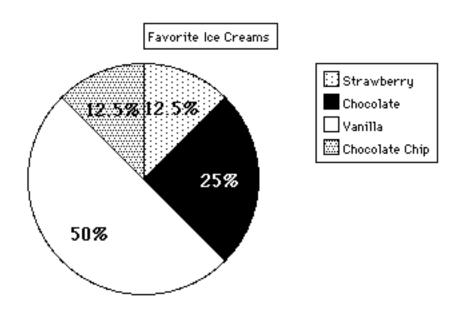
The instructor's explanation includes all the attributes described.

- 2. The instructor provides a set of data (Favorite Amusement Park Rides Activity Sheet) for the participants to generate a circle graph. Participants work in pairs to construct the graph, share their results with another pair of participants, and assess whether or not they have included all the attributes of a well-constructed circle graph.
- 3. The instructor provides a set of data (Favorite Chocolate Treat Activity Sheet) for each participant to generate a circle graph. Participants work individually to construct the graph and self-assess whether or not they have included all the attributes of a well-constructed circle graph.



FAVORITE ICE CREAM

<u>Flavor</u>	# of S	tudents	Frac	<u>tion</u>	Central Angle
Strawberry		3	3/24	or 1/8	45°
Chocolate		6	6/24	or 1/4	90°
Vanilla.		12	12/24	or 1/2	180°
Chocolate Chip		3	3/24	or 1/8	45°
	Total	24		24/24	360°
	Stu	<u>idents</u>	Frac	tion .	Central Angle

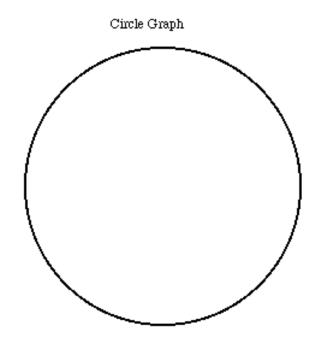




Favorite Amusement Park Rides

Use the information in the chart to make a circle graph of the favorite amusement park rides of the students surveyed.

Favorite Ride	Number of Students	Fraction	Central Angle
Sea Monster	16		
Twizzler	12		
Super Spin	8		
Water Log	6		
Wall Climber	6		
To	rtal		
	<u>Students</u>	<u>Fraction</u>	Central Angle



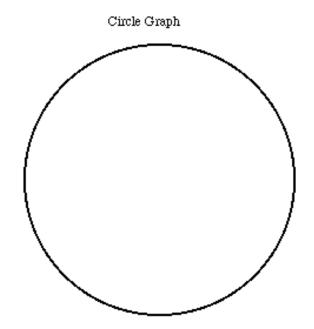
Explain what the graph tells you about the students' preferences for amusement park rides.



Favorite Chocolate Treat

Use the information in the chart to make a circle graph of the favorite chocolate treat of the students surveyed.

	<u>umber</u> f Students	Fraction	Central Angle
Chocolate Cake	6		
Chocolate Ice Cream	12		
Chocolate Chip Cool	sie 20		
Chocolate Candy	16		
Chocolate Milk	6		
Total			
	<u>Students</u>	<u>Fraction</u>	Central Angle



Explain what the graph tells you about the students' preferences for chocolate treats.



Activity: Frequency Distributions and Histograms

Format: Large Group/Individual/Small Group

Objective: Participants will analyze data by sorting, classifying, and

displaying it in frequency distributions and histograms.

Related SOL: 7.17, 8.12

Materials: One bag of counters, rulers, data collection activity sheet (Hand

Full), First Histograms Activity Sheet, Refined Histograms Activity Sheet, Attributes of Frequency Distributions and

Histograms Information Sheet

Time Required: 30 minutes

Directions:

1. The instructor explains the procedure that initiates the lesson.

Each participant will make an estimate of how many counters he/she can grasp in one hand from the bag of counters. Each participant will declare his/her estimate and all participants will write the number estimated in the Estimate column on their Hand Full Activity Sheets.

Each participant, in turn, will grasp as many counters as he/she can from the bag of counters, count the number of counters, and return the counters to the bag. The student will declare orally the number of counters grasped. All participants will write down the number grasped in the Actual column on the Hand Full Activity Sheets.

- Using the data in the Estimate column, participants count the number of pieces of data that belong to each interval in the frequency distribution for the estimates and record it in the Frequency column in the frequency distribution of the estimates.
- Using the data in the Actual column, participants count the number of pieces
 of data that belong to each interval in the frequency distribution for the
 actual number grasped and record it in the Frequency column in the
 frequency distribution of the actuals.
- 4. The instructor explains to the participants to construct bars on the First Histograms Activity Sheet. (Note: Graphs are not likely to accurately reflect all of the attributes of histograms that the instructor will next describe.)



5. The instructor explains the process that the participants experienced in making their first histograms from collecting data to putting the data in intervals to drawing a histogram. The instructor then defines and describes frequency distributions and histograms in terms of the way a statistician thinks.

A frequency distribution is a chart that shows the number of times that a particular measure or observation occurs.

The chart contains two columns. The first column lists all the measures (from highest to lowest) or observations. The second column gives the frequency, or number of times, that the measure or observation occurred.

Usually, the first step in making a frequency distribution is to list the possible measures or observations (first column) and then go through the data and make tally marks (second column) every time a measure or observation occurs. Then, the number of tally marks for each measure or observation is counted to find the frequency. Measures in a frequency distribution are usually grouped into intervals if the difference between the highest and lowest measures is 20 or greater.

To decide the size of an interval, the range (the difference between the highest and lowest measures) is divided by the desired number of intervals. If the quotient does not come out even, statisticians usually round it to the nearest odd number.

A histogram is a special type of bar graph in which the categories are equal ranges (intervals) of numbers and there are no spaces between the bars. The height of each bar is the numerical count of numbers in the range (interval).

The center of the horizontal axis is usually the midpoint of the intervals. It is customary to start with the lowest value on the left and proceed to the right with as many intervals as are necessary to include all the data. The horizontal axis does NOT need to begin at zero. An empty interval should be left at the lower and upper ends of the axis.

The vertical axis is the frequency of numbers in an interval. The vertical axis is marked off beginning with zero at the bottom and proceeding to the highest frequency. When statisticians graph frequency distributions, they use the "three-quarter-high rule" which means that the height of the highest bar is approximately three-fourths of the length of the horizontal axis. This rule prevents personal bias from influencing the height of the vertical axis. The vertical axis should be labeled "frequency" and the horizontal axis should be labeled to describe what is being measured.



The graph should have a descriptive title.

The instructor's explanation should have all the attributes described.

- 6. Following the instructor's explanation, the participants are given a blank Refined Histogram Activity Sheet and a written copy of the instructor's description of a frequency distribution and histogram, Attributes of Frequency Distributions and Histograms.
- 7. The participants work in pairs to construct a refined histogram using the frequency distribution of their estimates of the number of chocolate bars they could grasp.
- 8. When the pairs of participants have completed their histograms, they share them with other participants and assess whether or not they have included all the attributes of a well-constructed histogram.
- 9. The participants work individually to construct a refined histogram using the frequency distribution of the actual number of chocolate bars they grasped.
- 10. When the participants have completed their histograms, they share them with other participants and assess whether or not they have included all the attributes of a well-constructed histogram.



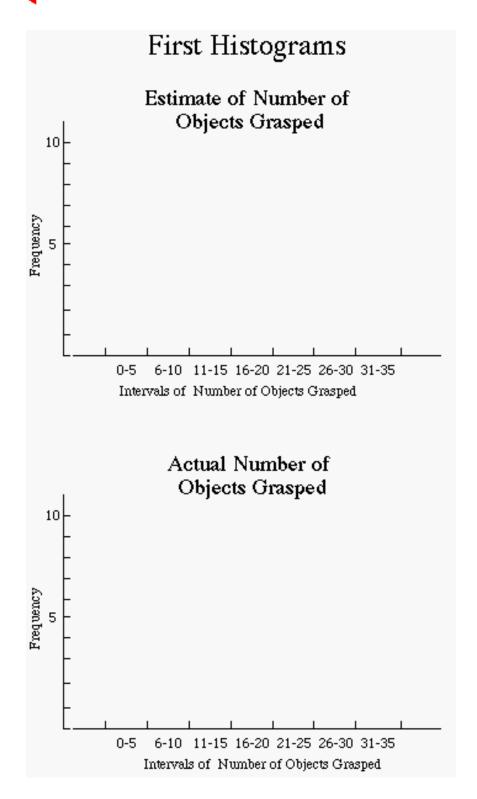


Number of Objects Grasped

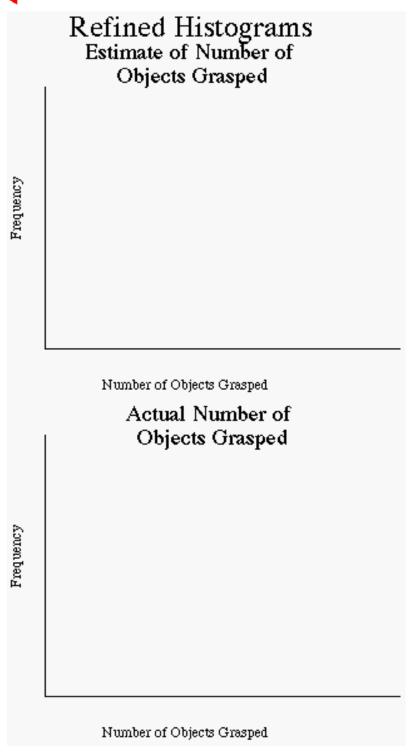
Student	Estimate	Actual

Freq	uency	
Distribution		
Estimate		
Interval	Frequency	
0-5		
6-10		
11-15		
16-20		
21-25		
26-30		
31-35		
Frequency		
Freq	luency	
-	uency ibution	
Distr	•	
Distr	ibution	
Distr Ac Interval 0-5	ibution ctual	
Distr Ac Interval 0-5 6-10	ibution ctual	
Distr Ac Interval 0-5 6-10 11-15	ibution ctual	
Distr Ac Interval 0-5 6-10 11-15 16-20	ibution ctual	
Distr Ac Interval 0-5 6-10 11-15 16-20 21-25	ibution ctual	
Distr Ac Interval 0-5 6-10 11-15 16-20	ibution ctual	
Distr Ac Interval 0-5 6-10 11-15 16-20 21-25	ibution ctual	











Attributes of Frequency Distributions and Histograms

- A frequency distribution is a chart that shows the number of times that a
 particular measure or observation occurs.
- The chart contains two columns. The first column lists all the measures (from highest to lowest) or observations. The second column gives the frequency, or number of times, that the measure or observation occurred.
- Usually the first step in making a frequency distribution is to list the possible measures or observations (first column) and then go through the data and make tally marks (second column) every time a measure or observation occurs. Then the number of tally marks for each measure or observation is counted to find the frequency. Measures in a frequency distribution are usually grouped into intervals if the difference between the highest and lowest measures is 20 or greater.
- To decide the size of an interval, the range (the difference between the highest and lowest measures) is divided by the desired number of intervals.
 If the quotient does not come out even, statisticians usually round it to the nearest odd number.
- A histogram is a special type of bar graph in which the categories are equal ranges (intervals) of numbers and there are no spaces between the bars. The height of each bar is the numerical count of numbers in the range or interval.
- The center of the horizontal axis is usually the midpoint of the intervals. It is customary to start with the lowest value on the left and proceed to the right with as many intervals as are necessary to include all the data. The horizontal axis does NOT need to begin at zero. An empty interval should be left at the lower and upper ends of the axis.
- The vertical axis is the frequency of numbers in an interval. The vertical axis is marked off beginning with zero at the bottom and proceeding to the highest frequency. When statisticians graph frequency distributions, they use the "three-quarter-high rule" which means that the height of the highest bar is approximately three-fourths of the length of the horizontal axis. This rule prevents personal bias from influencing the height of the vertical axis. The vertical axis should be labeled "frequency" and the horizontal axis should be labeled to describe what is being measured.
- The graph should have a descriptive title.



Activity: When It Rains

Format: Pairs

Objective: Participants will use their knowledge of line graphs to match

graphs with data sets.

Related SOL: 4.20, 5.18, 6.18, 8.12

Materials: When It Rains Activity Sheet

Time Required: 20 minutes

Background: A line graph is used to show changes over time for continuous

data. Points are plotted on the coordinate plane to represent change over time or any linear function. The units of division

on the axes are evenly spaced and plotted points are connected by line segments or dotted line segments.

Multiline graphs are used to compare two or more sets of

continuous data over time.

Directions:

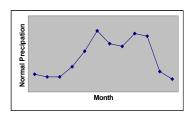
1. Distribute When It Rains Activity Sheet.

- 2. Have the participants match each line graph to its data set.
- 3. Have the pairs write a paragraph describing the analytical process used.

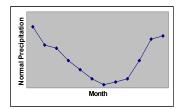


When It Rains

1.



2.



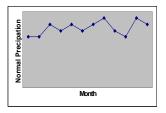
3.



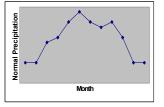
4.



5.



6.





When It Rains

NORMAL PRECIPITATION/SNOWFALL

(in centimeters)

	Kansas	New York	Fairbanks	Honolulu	Eureka	Miami
	City					
Jan.	4	8	3	12	20	7
Feb.	4	8	2	7	14	6
Mar.	8	10	2	9	13	6
Apr.	10	9	2	5	9	10
May	12	10	3	4	6	16
Jun.	15	9	5	2	3	24
Jul.	12	10	6	3	1	19
Aug.	11	11	7	3	2	18
Sep.	12	9	4	3	3	23
Oct.	9	8	3	5	9	22
Nov.	5	11	3	9	16	8
Dec.	5	10	3	10	18	4



Activity: Let the Graph Do the Talking

Format: Whole group

<u>Objectives</u>: Participants will develop skills in interpreting graphical

representations of data. They will discuss statistics that can be developed from graphs, compare and contrast data, find unique

and common features, describe trends and relationships between variables, and make predictions from the data.

Related SOL: 1.19, 2.23, 3.22, 4.20, 5.18, 6.18, 7.18, 8.12

Materials: Graph, data, and written summary cards for matching, example

graphs for discussion, and graphs for participants to analyze (types of graphs and level of analysis should vary depending on

the grade level of the participants), Graph 1 and Graph 2

Activity Sheets

Time Required: 10 minutes

<u>Background</u>: Once students have learned how to display data in graphs, it is

very important that they are able to summarize what they see in the graph. This interpretation should include drawing conclusions, comparing and contrasting, predicting, and examining relationships. Of course, the type of analysis that can be done depends on the data and the graph. Some examples used in these activities include the following:

A **bar graph** enables the researcher to compare how many subjects fall into specific categories. For example, a bar graph could show how many adults earned less than a high school education, earned a high school education, completed some college, and earned a college education. By examining the graph, determine how well the population is educated, comparing the number of people in each of the four groups.

A **scatterplot** enables the researcher to determine trends, make predictions, or see if there is a relationship between two variables. For example, a scatterplot could show the trend in median income of college graduates overtime. From this graph, use what happened historically to predict what might happen to income in the future.

A **line plot** enables the researcher to examine the distribution of a single variable. A line plot could illustrate the number of fat



grams in food purchased from a fast food restaurant. It shows how many foods had very few grams of fat versus many grams of fat. Draw a conclusion regarding how healthy the food is at such a restaurant.

A **box-and-whiskers** plot shows the researcher the spread and center of the data. These graphs are great tools for comparing two sets of data. Two box-and-whiskers plots could be used to illustrate the number of calories of various brands of ice cream and yogurt. Compare ice cream and yogurt from the graphs to determine which product tends to have more calories and which product has a larger variation in calories among the different brands. Being able to interpret graphs and data give students the ability for making decisions in their own lives.

Directions:

- 1. Start with a graph from the newspaper or the example graphs provided. Show the graph to the participants and ask the participants to describe what information and interpretation can be taken from the graph.
- 2. Lead this discussion with probing questions such as those given in the examples below.

Two Example Graphs and Leading Questions

Bar Graph of Births for each Month

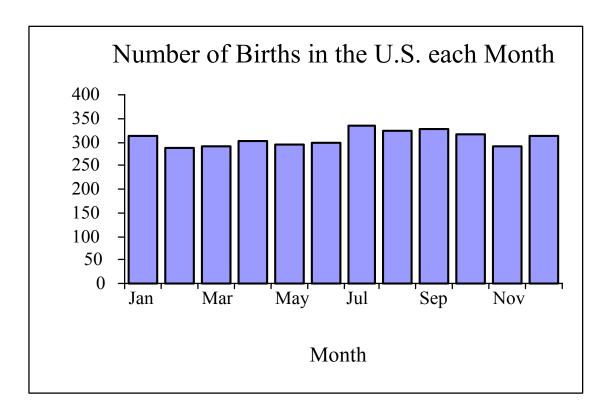
- Which months had the most number of births?
- Did one part of the year tend to have more births than another part of the year?
- How does the number of births in November compare to the number of births in July?
- Is there a trend during the year?

Line Plot of the Number of Days of Thunderstorms

- What was the most number of days of thunderstorms? the least number of days?
- Would you consider either of these points an outlier?
- What would you estimate to be the average number of thunderstorms for these cities?
- Is the data skewed left or right? What does this fact tell us about the distribution of thunderstorms in each city?



Example Graph 1

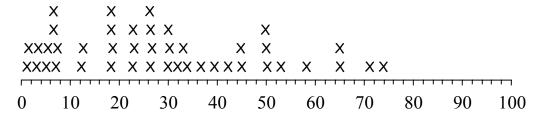


- Which months had the most number of births?
- Did one part of the year tend to have more births than another part of the year?
- How does the number of births in November compare to the number of births in July?
- Is there a trend during the year?



Example Graph 2

Number of Days per Year of Thunderstorms for Various Cities



Number of Thunderstorms

- What was the most number of days of thunderstorms? the least number of days?
- Would you consider either of these points an outlier?
- What would you estimate to be the average number of thunderstorms for these cities?
- Is the data skewed left or right? What does this fact tell us about the distribution of thunderstorms in each city?



Activity: Matching Game: Graphs, Data, Summary

Format: Large Group Matching to Form Groups of Three

Objectives: Participants will develop skills in interpreting graphical

representations of data. They will discuss statistics that can be developed from graphs, compare and contrast data, find unique

and common features, describe trends and relationships between variables, and make predictions from the data.

Related SOL 1.19, 2.23, 3.22, 6.18, 7.18, 8.12

Materials: Graph, data, and written summary cards for matching,

Matching Game Activity Sheet

Time Required: 15 minutes

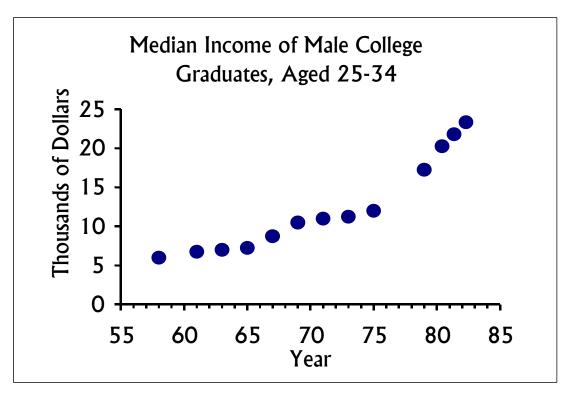
Directions:

1. Distribute the graph, data, and summary cards. Each participant should receive only one card. There are three cards that represent the same set of data. One card will have the raw data, a second card will have a graph of the data, and the third card will have a written summary of the data and graph. There are three sets of three cards representing the following data:

- Calories in Ice Cream and Yogurt
- Income for Male College Graduates
- Fat grams in Fast Food
- Level of Education for Adults
- Because there are three sets for each of the above, the participants cannot determine their match by just looking at the titles. The participants must sort out which data matches the graph and summary. There are a total of 36 cards. If there are fewer than 36 participants, remove cards in matching sets.
- 3. Use one matching set of cards as a demonstration. Show the data, discuss what type of graph would be appropriate for this data, provide an example graph that could be used for the data, and discuss the conclusions that can be drawn from the graph and the data, showing an example write-up of the data.
- 4. Participants should circulate around the room to find the two people who have cards that match their card.



Year	Income
58	6
61	6.8
63	6.9
65	7.3
67	8.7
69	10.4
71	10.9
73	11.3
75	12.1
79	17.2
81	20.3
82	21.1
83	21.9

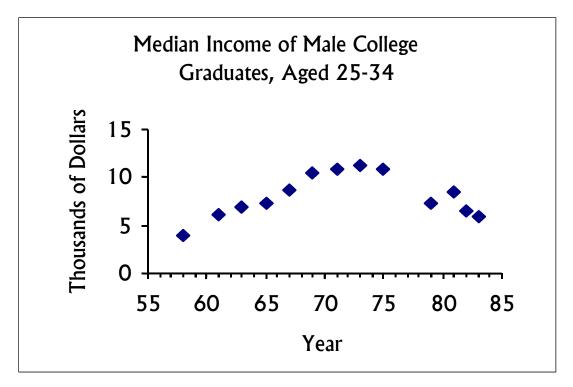




The graph illustrates the median income of college graduates, aged 25 - 34. It shows an upward trend in income over the past 25 years from 1958 to 1983. It appears that incomes rose slowly from 1958 to 1975 and then rose more rapidly from 1975 to 1983, with the largest one-year increase in the late seventies.

Income
4
6.2
6.9
7.3
8.7
10.4
10.9
11.3
10.9
7.3
8.5
6.5
6



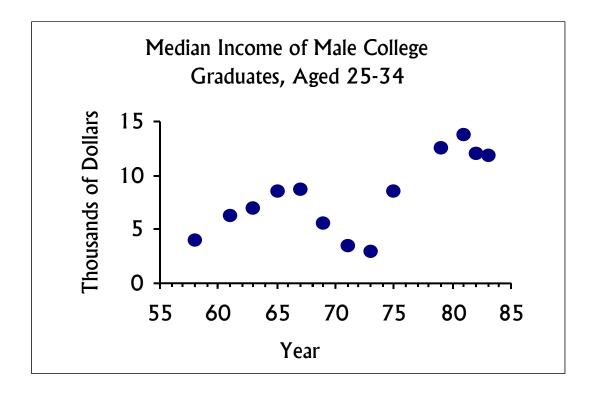


DATA-GRAPH-SUMMARY MATCH ACTIVITY

The graph illustrates the median income of college graduates, aged 25 - 34. It illustrates that income rose during approximately the first twenty years of this analysis from 1958 to 1973. After 1973, incomes began declining, dropping rapidly from 1975 and 1978.



Year	Income
58	4
61	6.2
63	7
65	8.5
67	8.7
69	5.5
71	3.5
73	3
75	8.5
79	12.5
81	13.8
82	12
83	11.8

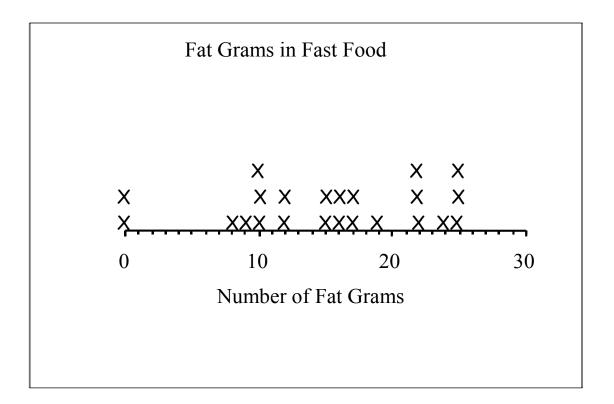




The graph illustrates the median income of college graduates, aged 25 - 34. The graph suggests that median income followed a cyclical pattern during the years from 1958 to 1983. Specifically, the median income rose steadily from 1958 to 1967, declined rapidly from 1967 to 1973, rose rapidly form 1973 to a peak of approximately \$15,000 in 1982 before leveling off in 1982 and 1983.

Fat Grams	in Fast Food
0	22
17	17
12	12
16	0
9	8
10	16
22	15
15	10
25	25
10	19
22	24
25	



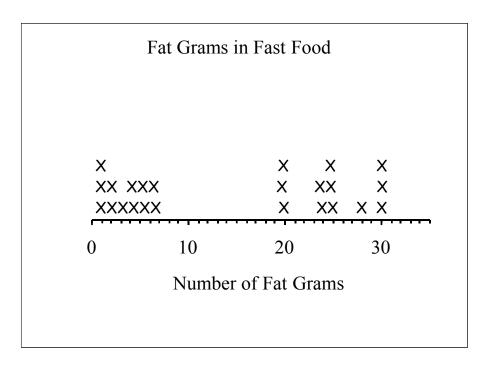


DATA-GRAPH-SUMMARY MATCH ACTIVITY

The data represent the number of fat grams in food purchased from a fast food restaurant. The graph shows that the amount of fat grams range from 0 grams to approximately 25 grams. There are a few items with zero grams of fat, most likely diet soda. However, most of the items have over 10 grams of fat.



Fat Gram	s in Fast Food
20	28
4	5
1	20
30	1
20	25
2	1
2	30
3	24
25	5
6	30
4	24
25	6

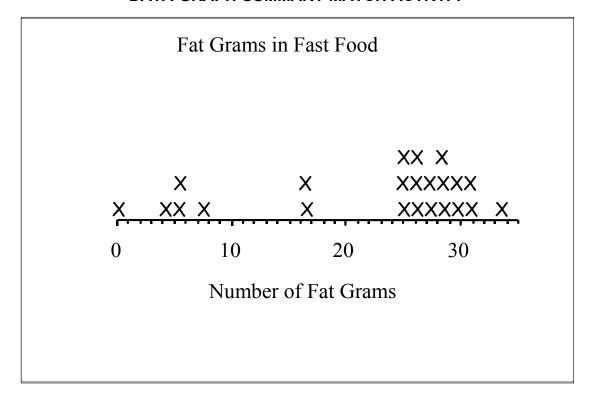




The data represent the number of fat grams in food purchased from a fast food restaurant. The graph shows that there appears to be two clusters of data, relatively healthy food with between 1 and 6 grams of fat and food that is not healthy with between 20 and 30 grams of fat.

Fat Grams	in Fast Food
31	27
27	30
16	4
0	28
26	5
16	30
7	5
25	25
26	28
28	25
26	29
31	34



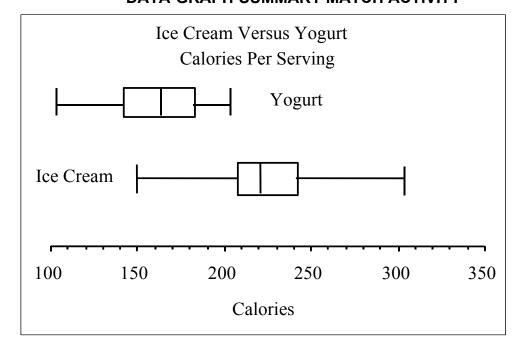


DATA-GRAPH-SUMMARY MATCH ACTIVITY

The data represent the number of fat grams in food purchased from a fast food restaurant. The graph shows that the amount of fat grams ranges from 0 grams to approximately 34 grams. There is a cluster of items around 5 to 8 grams but the largest cluster is between 25 and 31 grams. This cluster at the high fat contents implies that fast food is very high in fat grams.



Calories	per Serving
Ice Cream	Yogurt
150	100
150	120
175	125
200	130
205	140
205	150
210	155
215	155
215	160
220	165
225	170
225	170
230	175
240	180
250	180
275	185
300	190
305	200



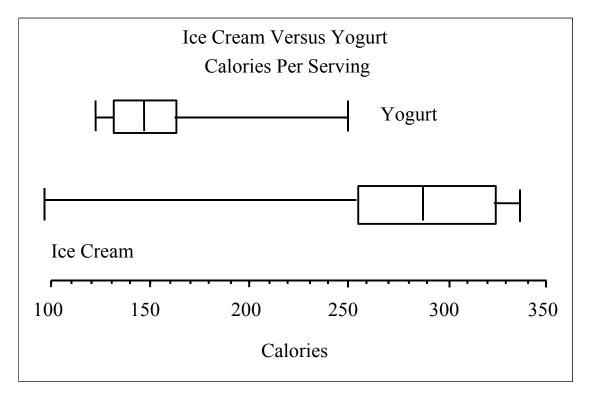


The data represent the number of calories in a serving of various brands of ice cream and yogurt. It appears that ice cream generally has more calories. Its median is approximately 220 calories compared to 160 calories for yogurt. Yogurt also has less variation in calories as seen in the lower range of values-yogurt ranges from a low of 100 calories to a high of 200 calories, a range of 100 calories, compared to ice cream which ranges from 150 calories to over 300 calories, a range of 150 calories.

DATA-GRAPH-SUMMARY MATCH ACTIVITY

Calories	per Serving
Ice Cream	Yogurt
100	120
150	125
180	125
250	125
255	130
260	135
275	135
275	140
275	140
300	150
300	150
310	155
325	155
325	160
325	170
325	180
340	190
340	250





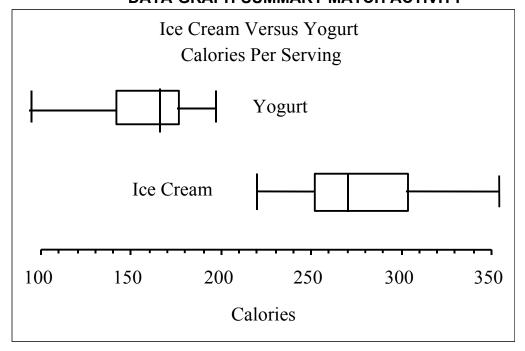
DATA-GRAPH-SUMMARY MATCH ACTIVITY

The data represent the number of calories in a serving of different brands of ice cream and yogurt. The graph illustrates that ice cream generally has more calories. Seventy-five percent of the ice cream brands have more than 250 calories, as seen by the fact that the box--the middle 50%--and right whisker--the top 75%-- is above 250 calories. The ice cream brands are skewed left with a high concentration in the high end of the scale. In contrast, the lower seventy-five percent of the yogurt brands is below 170 calories. In other words, the yogurt brands' calories are skewed right. Furthermore, the median number of calories for yogurt is approximately 150 calories compared to nearly 300 calories for ice cream. We can also see from the graph that ice cream brands have more variation in calories than yogurt.



Calarias no	· Comina
Calories per	•
Ice Cream	Yogurt
220	100
220	100
240	100
240	120
250	140
250	140
260	150
260	150
270	160
270	170
300	170
300	170
300	170
300	180
345	185
345	185
350	190
350	200

DATA-GRAPH-SUMMARY MATCH ACTIVITY



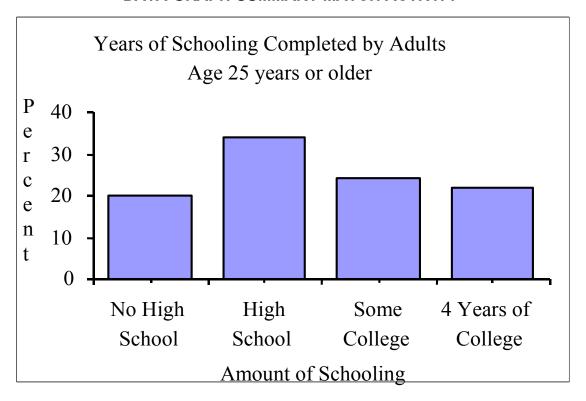


The data represent the number of calories in a serving of different brands of ice cream and yogurt. The graph illustrates that ice cream generally has more calories. In fact, all of the brands of ice cream examined have higher calories than all of the brands of yogurt selected. The maximum number of calories for yogurt is 200 calories compared to the minimum number of calories for ice cream is 220 calories. Furthermore, the median number of calories for ice cream is approximately 270 calories compared to 170 calories for yogurt. Both ice cream and yogurt have similar ranges and variation. Yogurt's range is approximately 100 calories and its interquartile range is approximately 50 calories. Similarly, ice cream's range is about 125 calories and its interquartile range is about 50 calories.

DATA-GRAPH-SUMMARY MATCH ACTIVITY

Years of Schooling	Percent
No High School	20
High School	34
Some College	24
4 Years of College	22





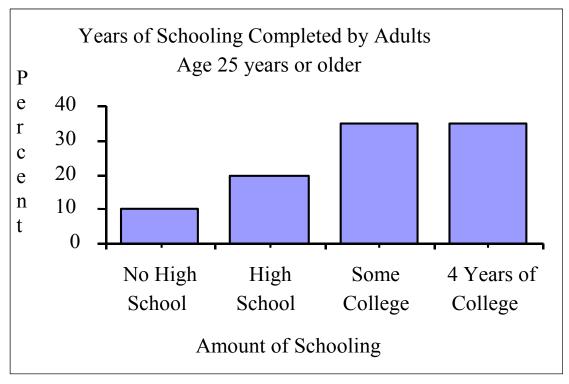
DATA-GRAPH-SUMMARY MATCH ACTIVITY

The data represent the percent of adults, age 25 and older, who have completed a certain level of schooling. Based on the graph, it appears that nearly equal percentages completed no high school, some college, and four years of college. The largest percentage was those adults who completed high school only.



Years of Schooling	Percent
No High School	10
High School	20
Some College	35
4 Years of College	35

DATA-GRAPH-SUMMARY MATCH ACTIVITY





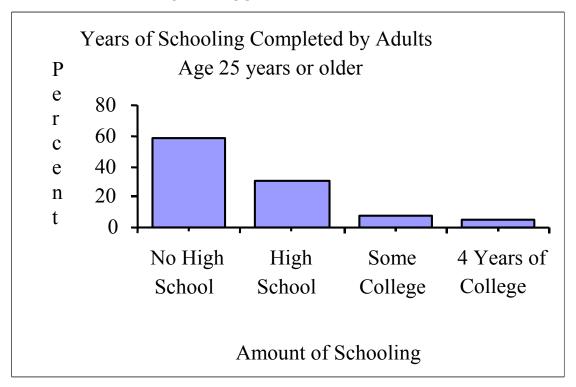
The data represent the percent of adults, age 25 and older, who have completed a certain level of schooling. Based on the graph, it appears that this population is fairly well educated with approximately 70 percent completing at least some college. Those completing no high school made up the smallest percentage at only 10 percent of the population.

DATA-GRAPH-SUMMARY MATCH ACTIVITY

Years of Schooling	Percent
No High School	58
High School	30
Some College	7
4 Years of College	5







The data represent the percent of adults, age 25 and older, who have completed a certain level of schooling. Based on the graph, it appears that the majority of this population has not completed high school. Those with some college and/or 4 years make up less than 15% of the entire population. These facts suggest that the population is not well educated.



Activity: Name That Graph

Format: Large Group/Small Group

Objective: Participants will understand how to choose a graphical method

that best displays a set of data.

Related SOL 4.20, 5,18, 6.18, 7.17, 8.12

Materials: Name That Graph Activity Sheet, Name That Graph Recording

Sheet

Time Required: 30 minutes

Directions:

1. Distribute the Name That Graph Activity and Recording Sheets to participants. Participants examine the various graphs and decide on a specific collection of data that exists in the room that could fit one of the graphs shown. An example may be the number of family members in each participant's home for graph 2.

- 2. The instructor should facilitate the actual collection of the data chosen by having participants raise hands or some other method. Participants decide if the graphical display chosen is accurate. Discuss what other graphs could have been chosen, if any, for the data.
- Encourage participants to work in small groups and come up with another set of data that will match a different graphical display. Participants collect the data and decide if their choice of graphs was accurate.
- 4. Collect Recording Sheets from each participant.
- 5. Lead a discussion with the large group.



Name That Graph

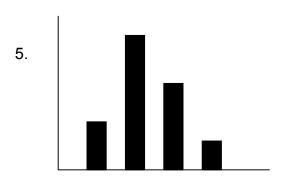


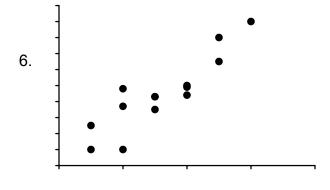


3.

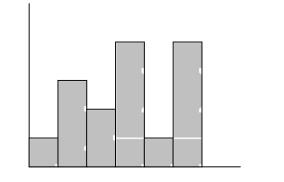
4.

0 3 3 5 7 8 9
1 0 2 3 5 6 6 8 9
2 0 1 3 3 3 5 5 8
3 0 5
4 5





7.



8.



Name That Graph Recording Sheet

Na	ame Da	ate
1.	What graphical display will best represent the data	a you plan to collect?
2.	Describe the data you will collect.	
Af	fter the Data Collection	
3.	Is the graphical display you chose appropriate for Why or why not?	the data you collected?
4.	Is there another graph that would be more approp collected? If so, what one would it be?	riate for the data you
5.	Choose another graphical display and describe the use for that display.	e data collection you would



Activity: Draw the Graph

Format: Small Groups (2-3 participants) and individual work

Objectives: Participants will develop skills in interpreting graphical

representations of data. They will discuss statistics that can be developed from graphs, compare and contrast data, find unique

and common features, describe trends and relationships between variables, and make predictions from the data.

Related SOL: 1.19, 2.23, 3.22, 4.20, 5.18, 6.18, 7.18, 8.12

Materials: Three different graphs for groups (types of graphs and level of

analysis should vary depending on the grade level of the

participants)

Time Required: 20 minutes

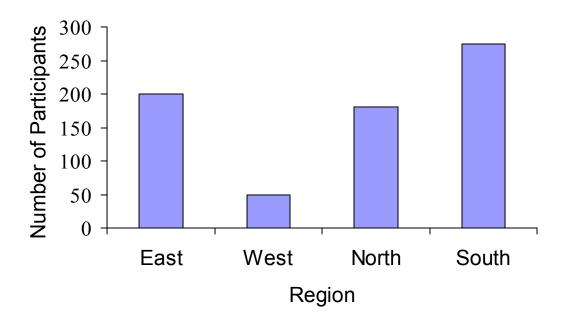
Directions:

- 1. Each individual participant in the group is given a different graph. Participants should not allow other members of their group to see their graph. Individually, the participants should write a summary of the graph using key words appropriate to the type of graph illustrated. The summaries should focus on comparing and contrasting categories, describing trends, identifying outliers and clusters. The summaries should not report numbers (such as, in 1990, there were 20 people and, in 1995, there were 300 people) or non-interpretative statements about the graph.
- 2. After each individual has written their summary, the group will play the game "Draw that Graph". In this game, there should be two "drawers" and one "analyzer". The analyzer will be the person whose graph is being drawn. The other two members of the group will be drawers. The analyzer reads his/her summary slowly to the group and the other two members attempt to recreate the graph. The analyzer cannot give specifics about where to draw what lines or points. Rather, he/she must rely on the written analysis. After the drawers are finished, the group can compare the graphs to the original graph and discuss what parts of the summary allowed them to graph the data and what could have been included in the summary to better explain the graph.



Graph 1

Number of Participants in Training Program

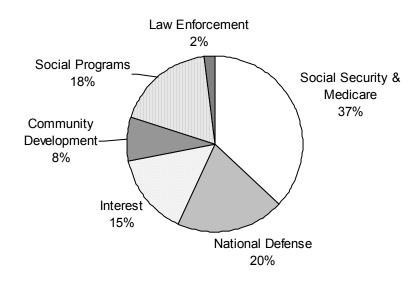


Summary:



Graph 2:

Federal Outlays

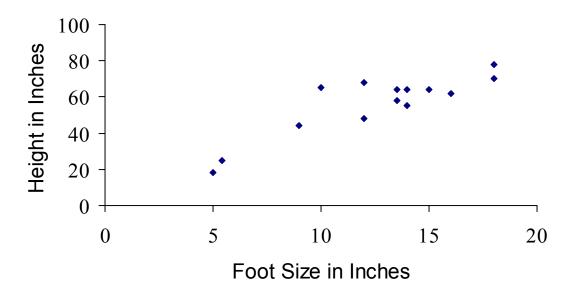


Summary:



Graph 3:

Foot Size versus Height



Summary:



Activity: Interpreting the Data

Format: Pairs, Small groups

Objectives: Participants will use the sample question information and

previous session work products to help them generate possible SOL assessment questions. Participants will learn which higher-order thinking skills are addressed in the sample

questions.

Related SOL: Reflection and review of previously discussed Probability and

Statistics SOL.

Materials: Copies of graphs from previous sessions, sample questions,

Name That Graph Activity Sheet, chart paper, and markers

Time Required: 30 minutes

Directions:

- 1. Participants work in pairs, using the sample questions and the graphs from the Name That Graph Activity Sheet. They discuss the data they collected and the graph they chose in relationship to the sample questions. They decide if they chose appropriate displays for the question they were answering. Were there other ways they could have organized the data to achieve similar results?
- 2. After some large group discussion about the sample questions and appropriate displays, the participants reorganize into grade level groups.
- 3. The instructor will have selected some of the graphs done in the previous sessions to use at this time. Enough copies of these graphs should be available to the appropriate grade level groups. Some possible graphs:

Session II Activities 6 and 7

Session III Activities 1, 2, 3, 4, 5, and 6

Session IV Activities 2, 3, and 4

- 4. Participants use these graphs to generate multiple-choice questions that could be asked on the SOL assessments. Questions should focus on interpretation and analysis of the graphs as well as alternative methods used to display data. For example: What would this bar graph look like as a stemand-leaf plot?
- 5. Each grade-level group writes at least two multiple-choice questions on chart paper for others to copy.



Sample Questions and Stems for Assessing Reasoning

Skill Questions

Abstracting What's the general pattern here?

Where else does it apply?

How can it be represented in another form (graphically,

etc.)?

Applying Using the principals of...as a guide, describe how you would

solve the problem.

Describe a situation that can be illustrated by using....

Analyzing List the main characteristics of...

Classifying What do the items have in common?

How else could the items be grouped?

Comparing Describe the similarities and differences between...

Compare the following two methods for...

How are these alike? How are these different?

Creating Make up a story describing what you see in this graph.

List as many ways as you can think of for describing...

Deduction What can you deduce from this data display?

Decision Making What would be the best? ...the worst?

Which one has the most? ...the least?

Error Analysis What specific errors have been made?

How can they be fixed?

Evaluating Describe the strengths and weaknesses of...

Using the given criteria, write an evaluation of...

Generalizing Formulate several valid generalizations from the data.

Induction What conclusions/generalizations can you draw from this?

What support do you have for these conclusions?

In light of the data presented, what is the most likely to

happen when...?

How would be likely to react to the following issue?



Justifying Which of the following alternatives would you favor and why?

Using the data, explain why you agree or disagree with the

following statement.....

Relating Cause

What are the major causes of ...?

and Effect

What would be the most likely effect on the data of ...?

Summarizing State the main points shown in the data display.

Briefly summarize the data displayed.

Synthesizing

Describe a plan for proving that....

Write a well-organized report that shows.....



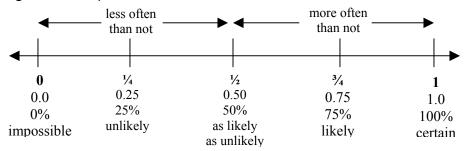
PROBABILITY Session 5

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Probability	Probability Background Information	155			
	Between 0 and 1	158	3.23, 4.19, 5.17	Recording Sheets, Probability Statements	Scissors
	Lay It on the Line	161	4.19, 5.17	Lay It On The Line Statements	
	What's In the Bag?	163	2.24, 3.23, 4.19, 5.17	What's In The Bag?	Paper bags, color tiles
	Fair or Not Fair	165	3.23, 4.19	Fair or Not? Game Sheet	Dice
	The Regatta	167	4.19, 5.17, 6.20, 7.14	Regatta Game Board, Log, Variations	Number cubes, 12 counters per participant, markers
	Tree Diagrams	178	6.20	Tree Diagrams	
	The Real Meal Deal	180	5.17, 6.20, 7.15	Real Meal Menu	Chart paper
	Reflections on the Course	183	All K-8 Probability and Statistics SOL		

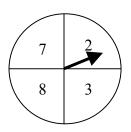


Probability Background Information

A measure used to express the likelihood of an event happening is considered **probability**. Probability can give you information about the likelihood of an event happening, but it is never a guarantee. The probability of an event happening can be expressed as a number from 0 to 1.



The probability of an event happening is the ratio of the number of positive outcomes to the number of possible outcomes. For example, suppose one wanted to know the probability of spinning a prime number on the spinner below. One could calculate the probability as follows:



P (prime number) =
$$\frac{3 \text{ prime numbers}}{4 \text{ numbers total}} = \frac{3}{4}$$

Ways to express probability:

There is a 3 out of 4 chance of this event occurring.

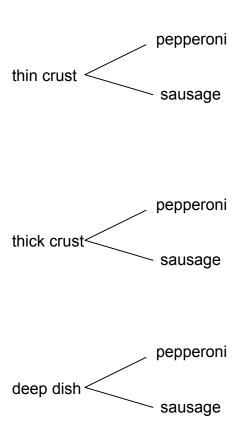
When conducting a probability experiment, data about the outcomes is being collected. This is called *sampling*. Sampling may not match the expected probability, but if conducted over extended periods of time, it should come fairly close. It is important to discuss with children the idea that what actually happens (experimental) doesn't always fit with what is expected to happen (theoretical).

In order to calculate the probability of an event, one may wish to create a sample space. A **sample space** lists all the possible outcomes of an event. For the above spinner, our sample space would be 2, 3, 7, 8. These are all of the possible outcomes of spinning the spinner.



Another way to determine the outcomes in a sample space is to draw a *tree diagram*.

Example: A pizza shop offers three styles of crust and two different toppings. How many different combinations of crusts and toppings are there?



There are 6 possible combinations. The probability of each combination occurring is 1/6. Therefore, the probability of a customer ordering a deep-dish pepperoni compared to all others is about 17%.

Simulation can be used to understand natural fluctuations or variation in data. For instance, if we were attempting to determine whether a spinner with 6 spaces is "fair", we might use simulation. A fair spinner would produce each outcome (1-6) an equal number of times over many, many spins. For instance, if we spun the spinner 60 times, we would expect each outcome to occur approximately 10 times. Notice we say approximately because we expect some variation – for instance, maybe eight 1s, thirteen 2s, etc. To determine if the spinner is fair, we need to know how much variation to expect—if we only got three 1s, is the spinner unfair or, in other words, is this variation too big to believe that the spinner is fair? By simulating outcomes that occur when we



make a selection randomly, we can better understand what is normal variation and what is abnormal variation.



Activity: Between 0 and 1

Format: Individual or small group

Objectives: Participants will classify statements as impossible, likely or

certain to help establish the concept of probability.

Related SOL: 3.23, 4.19, 5.17

Materials: Recording sheets, probability statements, scissors, glue or

tape, Between 0 and 1 Activity Sheet and Probability

Statements

Time Required: 20 minutes

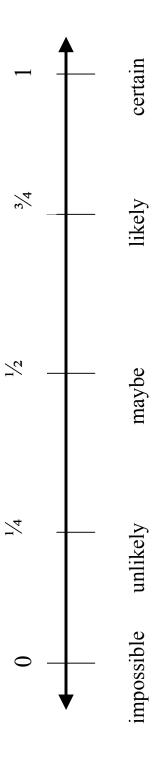
Directions:

1. Briefly discuss the vocabulary with the participants to gauge their understanding of the terms, impossible, likely, and certain.

- 2. Display the recording sheet on the overhead and discuss the number line concept.
- 3. Ask the participants, individually or in small groups, to assign values to the vocabulary words discussed. Record them on the transparency as participants record on their copy of the sheet.
- 4. Distribute the Probability Statements Activity Sheet. Have the participants cut the statements apart. Each statement should be placed on the number line recording sheet in the position corresponding to the likelihood of the event occurring.
- 5. When all of the number lines have been completed, ask groups members or individuals to share the results and discuss the similarities and differences.



Between 0 and 1





Between 0 and 1

Probability Statements

It will rain tomorrow.	You will have homework tonight.
Pizza will be served for lunch.	Your school has a principal.
The sun will rise tomorrow.	You will go to bed before 9:00 tonight.
You will have two birthdays this year.	You will go to Disney World sometime.
Your teacher is over 18 years old.	You will get tails when you flip a coin.
Two students will be absent tomorrow.	You throw a 4 on a die.
You will ride in a bus before the end of the school year.	Your teacher will let you have extra recess.
It will take you more than 1 hour to do your homework.	On your way to school you will see a live dinosaur.



Activity: Lay It on the Line

Format: Individual or small group

Objectives: Participants will classify statements as impossible or certain to

help establish the concept of probability.

Related SOL: 4.19, 5.17

Materials: Lay It on the Line Statements (1 per participant or group), glue,

scissors

Time Required: 20 minutes

<u>Background</u>: Referencing a number line should help participants understand

that the likelihood of an event occurring ranges from 0 to 1. It should also help them in evaluating the reasonableness of their calculations (an unlikely event should not have a probability of

0.90).

Directions:

- 1. Briefly discuss the vocabulary with participants to gauge their understanding of the terms. Display the "Between 0 and 1" overhead from the previous activity and discuss the number line concept with them.
- 2. Ask the participants, individually or in small groups, to assign values to these other probability words: probable, impossible, certain, maybe, always, unexpected, unlikely, possible, even chance, and improbable. Participants should record them on the recording sheet.
- 3. Have the students cut the statements apart. Each statement should be placed on the Lay It on the Line Recording Sheet in the position corresponding to the likelihood of the event occurring. When all of the number lines have been completed, post them and discuss the similarities and differences.



Lay It on the Line Statements

It will rain tomorrow.	Your teacher is more than 16 years old.
You will be given a homework assignment in math sometimes this year.	You will ride on a jet plane before the end of the year.
Drop a rock in water and it will sink.	You will have two birthdays this year.
Your school has a principal.	You will go to bed before 8:00 tonight.
Trees will sing in the afternoon.	You walk into the yard and see a live dinosaur.
You will learn to play the flute.	You will toss a die and show a 6.
The sun will rise tomorrow morning.	You will toss a coin and show a head.
You will go to Disneyland sometime.	Two students will be absent tomorrow.



Activity: What's In the Bag?

Format: Pairs

Objectives: Participants will conduct simple probability experiments to

predict outcomes.

Related SOL: 2.24, 3.23, 4.19, 5.17

Materials: Paper bags, color tiles, recording sheet

Time Required: 20 minutes

Directions:

1. Organize participants into pairs.

2. Give each pair a paper bag with 10 color tiles inside (7 blue and 3 red).

- Pairs will pull out one tile (without looking into the bag) and record the color on their recording sheet. The tile should be returned to the bag. Each pair will pull out tiles following this process a total of ten times
- 4. As pairs finish, have them record their results on a class graph at the front of the room.
- 5. When the class data is complete, have pairs look at the total number of blue and red tiles pulled and then make their prediction about the number of blue and red tiles in the bag.
- 6. After everyone has had a chance to predict, discuss the predictions and reasons why.
- 7. Have pairs look into their bags and record the actual results.
- 8. Discuss why their predictions may have differed from the actual number. What was helpful in making their predictions?
- 9. Repeat using bags with four colors of tiles. Discuss differences noted.



What's In the Bag?

Pick one tile from the paper bag. Record the color on the table below. Put the tile back into the bag. Choose another tile. Repeat this process 9 more times.

Blue										
Red										
My prediction: Actual results:										
There are		bl	ue tiles	3 .		blu	e tiles _.			_
There are		re	ed tiles			rec	l tiles _			_
Let's try a	ıgain wi	th four	colors!	!						
Blue										
Red										
Yellow	,									
Green										
My predic	tion:			·			Act	tual res	sults:	·
There are blue tiles.				blue tiles			_			
There are red tiles.			red tiles							
There are	ere are yellow tiles.				yellow tiles					
There are	!	g	reen til	es.		gre	en tiles	3		



Activity: Fair or Not Fair?

Format: Pairs

Objectives: Participants will be able to determine sample space and

fairness of a game.

Related SOL: 3.23, 4.19

Materials: 1 die per pair, Fair or Not Fair? Game/Recording Sheet (1 per

pair)

Time Required: 20 minutes

Background: The sample space of an experiment is nothing more than the

collection of all the possible outcomes for that experiment. In this case, all the possible results of the roll of a die are 1, 2, 3, 4, 5, 6. These are all of the possible outcomes of rolling a die – the sample space. A game is considered fair if the likelihood of

winning is the same as the likelihood of losing.

Directions:

1. Ask participants to predict the winner of each of the games described in the sheets distributed.

- 2. Have the participants pair up. (Ham and Cheese, Please! could be used for pairing.)
- 3. Each game should be played 20 times and the result of each game recorded.
- 4. Did the results match the prediction?
- 5. Was the game fair? Why or why not?
- 6. What could be done to make the game fair if it were not?
- 7. Have the participants construct the sample space to help in making the decision of how to make the game fair.



Fair or Not Fair? Game Sheet

Game Number	Player	What wins
1	Α	the numbers 1, 2, or 3
	В	the numbers 4, 5, or 6
2	Α	any odd number
	В	any even number
3	Α	any number less than 4
	В	any number greater than 4
		a 4 does not win
4	Α	any prime number
	В	any composite number

Fair or Not Fair? Recording Sheet

Game Number	Winner	Tally	Total
1	A: 1, 2, 3		
	B: 4, 5, 6		
2	A: odd numbers		
	B: even numbers		
3	A: numbers < 4		
	B: numbers >4		
4	A: prime numbers		
	B: composite numbers		



Activity: The Regatta

Format: Pairs or small group

Objectives: Participants will conduct simple probability experiments to

predict outcomes using a game format and two number cubes. Participants will investigate probability, sample space and tree

diagrams.

Related SOL: 4.19, 5.17, 6.20, 7.14

<u>Materials</u> Two number cubes per pair, The Regatta game board (1 per

pair), The Regatta transparency, 12 beans or small counters per pair, The Log Book Recording Sheet, large graph paper,

colored pencils or markers

Time Required: 40 minutes

<u>Background</u>: Participants will be conducting an experiment – any activity that

has two or more clearly discernible results or outcomes. As a result of the experiment, participants should be able to list the

sample space for that experiment. Sample space is a collection of all possible outcomes. Based on this sample space, participants should be able to determine the probability of an event occurring. Probability is defined as the ratio of the

number of favorable outcomes to all outcomes of an

experiment. An event is defined as any subset of the outcomes or any subset of the sample space – usually the outcome we

are looking for, a favorable outcome.

When all possible outcomes of a simple experiment are equally likely (each result is as likely to occur as every other), the theoretical probability of an event is

number of outcomes in the event number of possible outcomes

Experimental probability is based on the results of an experiment rather than a theoretical analysis of the experiment. Theoretical probability is based on a logical analysis of an experiment, not on experimental results.

The probability of an event is always 0, 1, or any number between 0 and 1. An impossible event has a probability of 0.



A certain event has a probability of 1.

Probabilities between 0 and 1 may be expressed as a ratio, a decimal, or a percent.

Directions:

Part I.

- 1. Explain to the participants that they will play a game using two number cubes or dice. Place The Regatta game board transparency on the overhead and place 12 counters on the starting line, one to represent each yacht.
- 2. Explain how the yachts move across the course. After a player rolls the cubes, he/she moves the yacht whose number is the sum of cubes ahead ONE space. For example, if a three and a five are rolled, the player moves yacht #8 one space forward.
- 3. Participants should predict which yacht would reach the finish line first. Record several responses. Have all participants predict, by a show of hands, which yacht they think will win.
- 4. Demonstrate the game on the overhead. Select two participants to model taking turns rolling the cubes and moving the appropriate yacht.
- 5. Pause frequently during the demonstration to look at the results. Ask participants if they notice any patterns in the way the yachts are moving. Ask participants to comment on why some of the yachts have not moved yet. Participants may want to predict again as the race progresses, changing their favorite as they watch the race. As new information is received, refinements in predictions are allowed.
- 6. Post the class graph and record the winning yacht.
- 7. Remind participants about accurate recording of their results.
- 8. Distribute The Regatta game boards, counters, and number cubes.
- 9. Have the participants begin playing The Regatta. Each race should be recorded on the class graph as soon as it is completed.
- Participants should play the game at least four times. All results should be recorded on the class graph so that a large amount of data can be collected for analysis in Part II.
- 11. Provide time for discussion of The Regatta game. Ask for participants to discuss any surprises they found in the results. Ask for "true statements" about the graph.



Part II.

- 1. Review the results displayed on the class graph. Ask participants to explain the results shown.
- 2. Lead the group in a discussion of the mathematics involved in The Regatta. Ask, "How many ways could you make a sum of six with two dice?" Remind them that this can include reversals, such as 4 + 2 and 2 + 4. For example, a green four and a white two is a different arrangement of the dice than a white four and a green two. On the overhead, record all the possible ways to make six using two dice.

Die 1	Die 2
1	5
2	4
3	3
4	2
5	1

- 3. List the ways to make several other numbers such as the number that won the most, or numbers that won few races.
- 4. On the overhead, show The Log Recording Sheet.
- 5. To help participants understand The Log Recording Sheet, color the dice on the transparency. For example, if participants used a red die and a green die, shade the dice in the horizontal row red and the dice in the vertical row green. Demonstrate how to fill in part of the chart by finding the sum and placing the sum in the appropriate box.
- Distribute The Log Recording Sheet to participants. Have them color the dice on The Log to match the ones they used when they played The Regatta. Have them complete The Log.
- 7. Check to see that participants have completed The Log correctly. Display a completed Log on the overhead.
- 8. Ask the participants to discuss any patterns they notice on The Log. Ask if The Log shows all of the ways to make sum six, as well as other sums. Ask the participants to discuss the frequency of obtaining each sum. How does this frequency relate to the winners of The Regatta?
- 9. Ask the participants how many ways a seven can be made. From a review of The Log, they should determine that there are six ways to make a seven. How many possible sums can be obtained by rolling two dice? From The Log, there are 36 possible outcomes when two dice are rolled.



10. Have the participants use the chart to show numerical ways to represent probabilities. One way of representing the probability of each number 1 through 12 is shown below:

```
a sum of 7:
                  6 out of 36
                                or 1 out of 6
                                                   or 1/6
a sum of 6 or 8: 5 out of 36
                                or 1 out of 7
                                                      1/5
                                                   or 1/9
a sum of 5 or 9: 4 out of 36
                               or 1 out of 9
a sum of 4 or 10: 3 out of 36
                               or 1 out of 12
                                                  or 1/12
a sum of 3 or 11: 2 out of 36
                                or 1 out of 18
                                                  or 1/18
a sum of 2 or 12: 1 out of 36
                                or 1/36
a sum of 1:
                 0 out of 36
                                or 0
```

These fractional probabilities can be used to remind participants of the relationship of a written number to the real world.

Part III.

1. Give each pair a calculator and have them calculate the experimental probability of each winner in The Regatta. Then have them calculate the theoretical probability of each sum listed on The Log. A comparison of the two probabilities should be made.

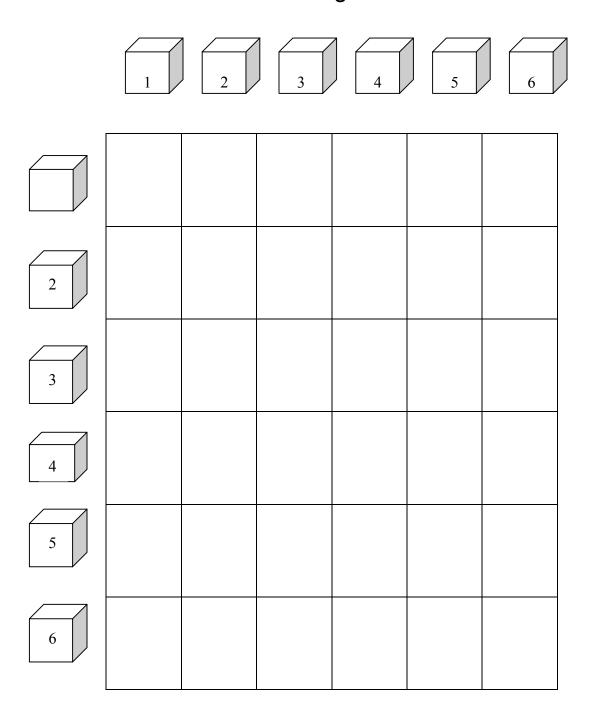


The Regatta Game Board

1		Ů,	1
2			2
3		Ĵ	3
4		J.	4
5		Ĵ.	5
6		L	6
7		Ĵ	7
8		Ţ	8
9		Ĵ	9
10		Ĵ.	10
11		Ĵ.	11
12			12



The Log





Variation for Part II.

- 1. Lead the group into the development of the *sample space* all possible outcomes. Record all the sums previously identified on the chalkboard or overhead.
- 2 3 4 5 6 7 8 9 10 11 12
- 2. Ask "How many ways can I get a sum of 2?" ... "of 3?", and so on. Continue until all sum combinations are found. Record each response.

At this point, ask the participants if they see a pattern? If so, predict how many combinations are there to get a sum of 7? Record the predictions. Then, verify the number of combinations.

Ask "Can you predict how many combinations to get a sum of 8?" Record the prediction. Verify the number of combinations.

Participants may suggest (1,7); that's okay, remind them that number cubes have only 1 to 6 to work with.

Ask "What happened to our prediction? What went wrong?"



Do you want to predict how many combinations there are for 9? Record any predictions. Verify the combinations.

Continue in a similar fashion until the entire sample space is listed.

How many possible outcomes are there?

NOTE: This is a good time to construct a tree diagram and show participants how to determine the sample space with a tree diagram.

Now, which sum do you think should occur most often? Does this match your prediction, or the results shown on our graph? Why or why not?



Variation for Part I.

Ask the participants to predict the winner of each of the following games.

Rule of play:

Two dice are rolled; the sum of the faces of the two dice is computed.

Game	Player	What wins:	
Number			
1	Α	Even sums	
	В	Odd sums	
2	Α	Sums of 2, 3, 4, 5, 6	
	В	Sums of 8, 9, 10, 11, 12	
3	Α	Sums that are prime	
	В	Sums that are composite	
4	Α	Sums > 7	
	В	Sums < 7	
5	Α	Sums of 4, 5, 6, 7	
	В	Sums of 2, 3, 10, 11, 12	

Pair the participants. Have each pair pick a game to play. Be sure all games are played by at least one pair. Have the pairs roll 50 times; one player rolls 25 times, while the other tallies and records the winner. After 25 rolls, have the players switch.

After the tallies are complete, combine all the data into one class chart and discuss the results.

Possible questions:

Do you think the game you played was fair or not? If it was fair, why? If it was not fair, why?

What could be done to make the game fair?



Variation for Part I: Two Dice Difference

How to Play: The game is played by 2 participants, each of whom has one sixsided die. Participants roll simultaneously and find the absolute (or positive) value of the difference of the numbers.

A wins if the value is 0, 1, or 2.

B wins if the value is 3, 4, or 5.

Pairs roll the dice 10 times and keep a record of wins and losses.

Collect class date.

Is the game fair? Why or why not?

Have the participants construct the sample space and calculate the theoretical probability of each outcome.

Based on the sample space, how could the game be made fair?

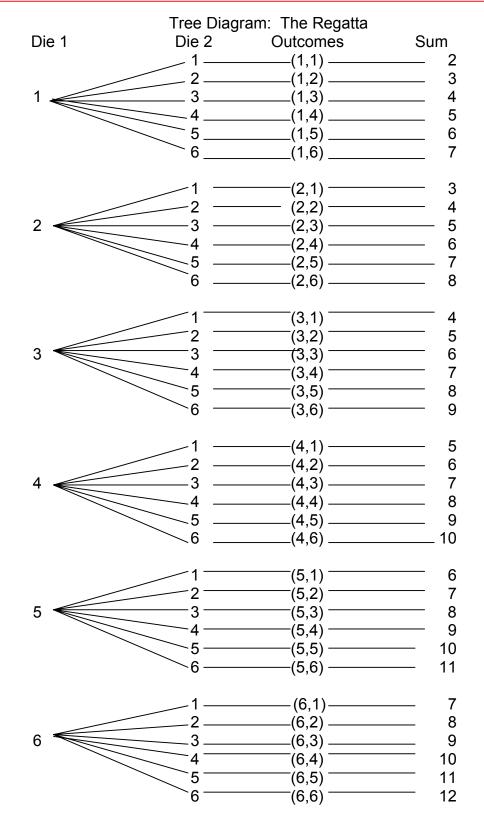
Two Dice Sum: Fair Game Recording Sheet

Use the line plot to tally the sums that were rolled.



Game Number	Winner	Tally	Total
1	A: even sums		
	B: odd sums		
2	A: sum of 2, 3, 4, 5, 6		
	B: sum of 8, 9, 10, 11, 12		
3	A: prime sum		
	B: composite sum		
4	A: sum >7		
	B: sum <7		
5	A: sum of 4, 5, 6, 7		
	B: sum of 2, 3, 10, 11, 12		







Activity: Tree Diagrams

Format: Pairs or individually

Objectives: Participants will construct a tree diagram.

Related SOL: 5.17, 6.20

Materials: Tree Diagrams Recording Sheet

Time Required: 20 minutes

Directions:

1. Choose three participants to come to the front of the room. Try to choose people who are wearing different types of outfits.

- 2. Construct a tree diagram (as a total group) of all the possible combinations of outfits that can be made from the clothes the participants are wearing. For example: blue shirt (person 1), jeans (person 2), tennis shoes (person 3).
- 3. Continue with the Tree Diagrams Activity Sheet on constructing tree diagrams from pizza choices.
- 4. Discuss how the sample space changes when you add additional choices.





Tree Diagrams

You are trying to decide which pizza to order for dinner. Your choices for crust are: regular, thin, and deep dish. You only want one topping and will either choose pepperoni or sausage.

Construct a tree diagram to show the possibilities you have from which to choose one crust with one topping.

How would your sample space change if you added bacon as a third topping choice?



Activity: The Real Meal Deal

Format: Pairs

Objective: Participants will use a tree diagram and the Fundamental

Counting Principle to determine the sample space of an event.

Related SOL: 5.17, 6.20, 7.15

Materials: Real Meal Restaurant Menu, chart paper for tree diagrams

Time Required: 20 minutes

Background: The Fundamental Counting Principle is a method for finding the

number of ways that two or more events can occur by

multiplying the number of ways that each event can occur. The Principle states that, if successive choices are made, then the total number of choices is the product of the number of choices

at each stage.

For example, if you have 3 shirts and 2 pairs of jeans, then you have a total 6 different outfits to wear. Each shirt may be worn with each pair of jeans. There are 3 shirts times 2 pairs of

jeans for a total of 6 outfits.

A tree diagram is a visual way to see all of the outcomes.



Directions:

- 1. Based on the menu of the Real Meal Restaurant, participants will use the Fundamental Counting Principle to determine the number of different meals that can be served.
- 2. Based on customer wishes, participants will determine and display the choices using a tree diagram.



NOTE: This may be a good time to have participants go back to The Regatta and develop a tree diagram for the sample space formed by finding the sum of two dice.



REAL MEAL RESTAURANT

SANDWICHES: Ham and Turkey Club Hamburger

Rachael on Rye Deli Cold Cut Special

Sliced BBQ Pork BLT

FRENCH FRIES: small medium large

SALADS: Garden Salad <u>DRESSINGS</u>: Ranch

Chef Salad French

Cobb Salad Creamy Italian

BEVERAGES:

Soft Drinks: small medium large

Coke Pepsi Sprite

Tea: medium large Coffee: medium large

Milk: regular low-fat

- 1. How many possible meals can be served at the Real Meal, choosing only one item from each category?
- 2. How many choices are there if a customer wants the following:
 - a. a soft drink, sandwich, and fries? Display the choices with a tree diagram.
 - b. a sandwich, fries, and milk? Display the choices with a tree diagram.
 - c. salad with dressing and tea? Display the choices with a tree diagram.
 - d. a sandwich, salad with dressing, and coffee? Display the choices with a tree diagram.



Activity: Reflections on the Course

Format: Pairs, Large Group

Objective: Participants will discuss what they have learned and how they

plan to use what they have learned in their teaching.

Participants will make a commitment to revising their instruction

of Probability and Statistics, where appropriate.

Related SOL: All Probability and Statistics SOL

Materials: Paper and pencil

Time Required: 20 minutes

Directions:

- 1. Have participants work in pairs to discuss what they will do differently when teaching probability and statistics.
- 2. In a round-robin session, have participants share their future plans for teaching probability and statistics.